

Mathematical methods, models and modelling (MST210) content listing

Unit 1 <i>First- and second-order differential equations</i>	Analytic solution of first-order differential equations using either separation of variables or the integrating factor method. Direction fields and numerical solution by Euler's method. Solution of linear constant-coefficient second-order differential equations. Using the method of undetermined coefficients to find particular integrals for simple inhomogeneous differential equations.
Unit 2 <i>Vector algebra and statics</i>	Vectors both geometrically and algebraically. Scalar and vector products. Modelling forces (weight, normal reaction, tension, friction) as vectors. Calculating torques. Application to static equilibrium problems.
Unit 3 <i>Dynamics</i>	Newton's laws of motion. Sliding friction. Models of air resistance. Application to single particle dynamics problems. Projectile motion.
Unit 4 <i>Matrices and determinants</i>	Solving systems of linear equations using Gaussian elimination. Matrices as linear transformations. Matrix algebra and matrix inversion. Evaluating determinants.
Unit 5 <i>Eigenvalues and eigenvectors</i>	Finding eigenvalues and eigenvectors by hand for simple cases. Iterative methods for finding eigenvalues of large matrices.
Unit 6 <i>Systems of linear differential equations</i>	Solving first-order and second-order systems of linear differential equations by using the eigenvalue and eigenvectors of the coefficient matrix.
Unit 7 <i>Functions of several variables</i>	Visualising functions of several variables, contour maps and gradient along a path. Partial derivatives. The gradient vector. Taylor polynomials. Classifying stationary points.
Unit 8 <i>Mathematical modelling</i>	Case study of pollution modelling in the Great Lakes. Overview of the modelling process. Dimensional consistency.
Unit 9 <i>Oscillations and energy</i>	Modelling forces due to a spring (Hooke's law). Motion of a single particle under influence springs. Potential energy. Principle of conservation of mechanical energy and equivalence with Newton's laws. Application of energy conservation to mechanical systems.
Unit 10 <i>Forcing, damping and resonance</i>	Modelling dampers. Single particle systems with springs and dampers. Forced oscillations and resonance.
Unit 11 <i>Normal modes</i>	Analysing mechanical systems with two or more particles with springs.
Unit 12 <i>Systems of differential equations</i>	Modelling interacting populations using non-linear differential equations. Qualitative solutions via phase plane portraits. Equilibrium points. Using linearization to classify equilibrium points.
Unit 13 <i>Fourier series</i>	Periodic functions. Representing periodic functions as Fourier series of sines and cosines. Pointwise convergence theorem.
Unit 14 <i>Partial differential equations</i>	Separation of variables applied to partial differential equations. Application to the wave equation and also to the heat equation.
Unit 15 <i>Vector calculus</i>	Scalar and vector fields. Gradient of a scalar field. Cylindrical and spherical coordinates.
Unit 16 <i>Further vector calculus</i>	Divergence and curl of a vector field. Line integrals of scalar and vector fields. Conservative fields and the curl test.
Unit 17 <i>Multiple integrals</i>	Area and volume integrals in Cartesian coordinates. Area integrals in polar coordinates. Volume integrals in cylindrical and spherical coordinates.
Unit 18 <i>Reviewing the model</i>	Dimensional analysis. Evaluating mathematical models. Case study on Great Lakes revisited.
Unit 19 <i>Systems of particles</i>	Centres of mass of systems consisting of particles, simple geometric objects and laminas. Analysing collision problems using conservation of momentum and Newton's law of restitution.
Unit 20 <i>Circular motion</i>	Uniform and non-uniform motion in a circle. Defining angular velocity and angular momentum.
Unit 21 <i>Rotating bodies and angular momentum</i>	Analysing the motion of mechanical systems that rotate about an axis that may accelerate (but has a fixed direction).