***Optimization* (M373)** content listing

Introductory Unit	
Block 1 Unit 1 *Introduction to iterative methods*	Basic definitions related to optimization problems including types of error and absolute and relative error; numerical methods for finding roots of functions of one variable: bisection method, simple iteration, the Newton-Raphson method, the add-nx method; contraction mapping theorem; rate of convergence; stopping criteria.
Block 1 Unit 2 *Systems of linear equations*	Elementary matrices; methods for solving systems of linear equations: Gaussian elimination, LU decomposition, Jacobi and Gauss-Seidel methods; vector and matrix norms; guarantee and rate of convergence for iterative methods; absolute and relative error; stopping criteria.
Block 1 Unit 3 *Ill-conditioning and induced instability*	Absolute and relative ill- and well-conditioning of problems; induced instability; discussed in the context of finding roots of a function of one variable and solving systems of linear equations; background material: partial derivatives, Taylor polynomial approximations; absolute and relative condition numbers; p-figure arithmetic; Gaussian elimination with partial pivoting; Gaussian elimination with iterative refinement.
Block 1 Unit 4 *Systems of non-linear equations*	Methods for finding roots of functions of n variables: Jacobi and Gauss-Seidel methods, simple iteration, the Newton-Raphson method, the add-Nx method; methods for finding an initial estimate: contour plot, grid search; the contraction mapping theorem; rate of convergence; stopping criteria; absolute and relative condition numbers.
Block 1 Unit 5 *Mathematical modelling*	Stages of the mathematical modelling process; case studies; local and global minima and minimizers; finding and classifying stationary points; the least squares method to evaluate a model; dimensional consistency; sensitivity analysis; revising the model.
Block 2 Unit 1 *Linear programming – the basic ideas*	Formulation of linear programming models; general form, standard form and canonical form; main, slack and surplus variables; feasible regions; types of solution; graphical solution of two-dimensional linear programming models; algebraic and matrix versions of the Simplex method; Simplex method termination theorem.
Block 2 Unit 2 *Linear programming* *– the two-phase simplex method*	Primal model; shadow price and reduced cost vectors; dual model; duality theory; the two-phase Simplex method; effect of changes in the data; computational refinements.
Block 2 Unit 3 *Integer programming*	Integer programming models and associated linear programming models; the branch-and-bound method; 0-1 variables; set-covering problems; set-packing problems; knapsack problems; piecewise linear relationships; reducing computational resource.
Block 2 Unit 4 *Applications of linear and* *integer programming*	Formulating larger models; multi-objective optimization; the travelling salesman problem, the machine scheduling problem.
Block 3 Unit 1 *Minimizing a function of one or two variables*	Methods for minimizing a function of one variable: finding and classifying stationary points, the Newton-Raphson method for finding a local minimizer, interval reduction methods (grid search, golden section search); methods for minimizing a function of two variables: finding and classifying stationary points, line searching: alternating variables method, steepest descent method; stopping criteria; sensitivity analysis.
Block 3 Unit 2 *Unconstrained non-linear optimization*	Local and global minima and minimizers of functions of n variables; sufficient conditions for a local minimizer; background material: positive definiteness, conjugacy; methods for minimizing a function of n variables: the Newton-Raphson method, the Newton-Raphson method with line searches, the rank one method, the Broyden-Fletcher-Goldfarb-Shanno method, the Fletcher Reeves method; evaluation and comparison of the methods; least squares problems and the Gauss-Newton method; error analysis; stopping criteria; sensitivity analysis.
Block 3 Unit 3 *Constrained non-linear optimization*	Mathematical theory for equality-constraint models; Lagrangian conditions; methods for solving equality-constraint models: the quadratic penalty-function method, the Newton-Lagrange method, the augmented Lagrangian method; mathematical theory for inequality-constraint models including Karush-Kuhn-Tucker conditions; methods for solving inequality-constraint models: quadratic penalty-function method, the augmented Lagrangian method; sensitivity analysis.
Block 3 Unit 4 *Optimization problem-solving*	The mathematical modelling cycle applied to a number of case studies using the methods of the previous Units.