

Complex analysis (M337) content listing

Unit A1 <i>Complex numbers</i>	Complex numbers properties and arithmetic The complex plane, modulus and argument, and polar form Finding roots of complex numbers and solving quadratic equations Sketching sets in the complex plane The Triangle Inequality
Unit A2 <i>Complex functions</i>	Complex functions properties Standard complex functions, and methods for combining functions Paths in the complex plane Standard paths Exponential, trigonometric, hyperbolic and logarithmic functions
Unit A3 <i>Continuity</i>	Sequences of complex numbers, and rules for testing for convergence or divergence Continuous functions – sequential definition and ϵ - δ definition Limits of functions Open sets, closed sets, path connected sets, regions, bounded sets, compact sets and their properties
Unit A4 <i>Differentiation</i>	Derivatives and their properties The Cauchy–Riemann equations Smooth paths and angles between smooth paths Conformal functions
Unit B1 <i>Integration</i>	Revision of real integration Integrating complex functions along smooth paths and contours The Fundamental Theorem of Calculus and Integration by Parts Estimating contour integrals
Unit B2 <i>Cauchy's Theorem</i>	Simple contours and simple-closed contours The Jordan Curve Theorem Cauchy's Theorem and Cauchy's Integral Formulas Liouville's Theorem and the Fundamental Theorem of Algebra Methods for evaluating contour integrals The Primitive Theorem and Morera's Theorem
Unit B3 <i>Taylor series</i>	Series of complex numbers, and rules for testing for convergence or divergence Power series The radius of convergence The Differentiation and Integration Rules for Power Series Taylor series and Taylor's Theorem Taylor series of standard functions Rules for calculating and manipulating Taylor series The Uniqueness Theorem.
Unit B4 <i>Laurent series</i>	Isolated singularities, removable singularities, poles and essential singularities Laurent series and Laurent's Theorem Calculating Laurent series Classifying the behaviour of functions near singularities The Casorati–Weierstrass Theorem The residue of a function at a singularity Evaluating integrals using Laurent series
Unit C1 <i>Residues</i>	Methods for calculating residues Cauchy's Residue Theorem Calculating real trigonometric and improper integrals using the Residue Theorem Summing series using the Residue Theorem Analytic continuation More applications for calculating improper integrals
Unit C2 <i>Zeros and extrema</i>	Winding numbers The Argument Principle and Rouché's Theorem The Open Mapping Theorem, the Local Mapping Theorem, the Inverse Function Rule Finding maxima and minima using the Maximum and Minimum Principles Schwarz's Lemma Uniform convergence of sequences and series Weierstrass' M-test and Weierstrass' Theorem The zeta function and gamma function

Unit C3 <i>Conformal mappings</i>	Properties of linear functions and the reciprocal function The extended complex plane, the point at infinity, generalised circles, the Riemann sphere and stereographic projection Möbius transformations – definitions and properties. Images of generalised circles, generalised open discs and lunes under Möbius transformations Images of regions under conformal mappings. Composing conformal mappings
Unit D1 <i>Fluid flows</i>	A model for fluid flow governed by a velocity functions Streamlines and stagnation points for the flow Circulation and flux. Ideal flows A source, sink and vortex of a flow Complex potential functions and stream functions Examples of fluid flows The Joukowski function and its properties The Obstacle Problem, and how to solve this problem for certain obstacles using the Flow Mapping Theorem Flow past and an aerofoil
Unit D2 <i>The Mandelbrot set</i>	Iterating functions and iteration sequences Classifying types of fixed points of functions Conjugate functions and conjugate iteration sequences Complex quadratic functions The escape set and keep set Periodic points, cycles and multipliers Graphical iteration of real functions Connected and disconnected sets The Mandelbrot set and its properties The Fatou–Julia Theorem Methods for determining whether a point lies in the Mandelbrot set Saddle-node and period-multiplying bifurcations.