MST124 sample iCMA

This document illustrates the style of questions in MST124 iCMAs.

Questions

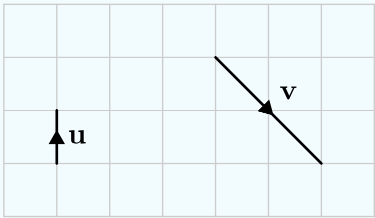
1 2 3

Question 1

Not yet answered

Marked out of 1.00

The vectors $\mathbf{u}$ and $\mathbf{v}$ are illustrated below.



Complete the following statement about the vector $\mathbf{w} = 3\mathbf{u} + \mathbf{v}$.

The tip of $\mathbf{w}$ is ____ unit(s) to the right and ____ unit(s) above the tail of $\mathbf{w}$.

Questions

1 2 3

Question 2

Not yet answered

Marked out of 1.00

Given that

$$2(\mathbf{a} - \mathbf{b}) + 3(\mathbf{x} - \mathbf{a}) = 2\mathbf{b},$$

express $\mathbf{x}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(Type `a` to enter $\mathbf{a}$ and to enter `b` type $\mathbf{b}$.)

$\mathbf{x} =$ ____

Questions

1 2 3

Question 3

Not yet answered

Marked out of 1.00

A ship sails on a course with bearing 60° for 2 miles, then turns and sails 3 miles on a bearing of 310°.

What is the distance (in miles, to one decimal place) and the bearing (to the nearest degree) of its final location from its initial location?

The distance is ____ miles (to 1 d.p.).

The bearing is ____ ° (to the nearest degree).

Solutions

Questions

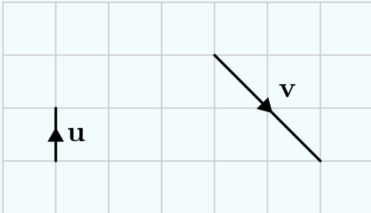
1 2 3

Question 1

Correct

Mark 1.00 out of 1.00

The vectors **u** and **v** are illustrated below.

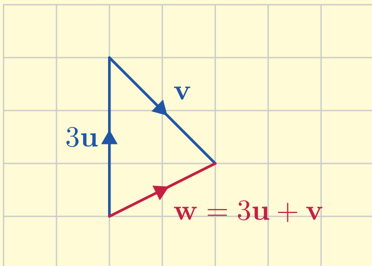


Complete the following statement about the vector $\mathbf{w} = 3\mathbf{u} + \mathbf{v}$.

The tip of **w** is [2] unit(s) to the right and [1] unit(s) above the tail of **w**.

Your answer is correct.

The vector **w** can be obtained from **u** and **v** using the triangle rule for vector addition, and the rule for scalar multiples of vectors, as shown below.



So, the tip of **w** is 2 units to the right and 1 unit above the tail of **w**.

See Unit 5, Subsection 5.1.

Questions

1 2 3

Question 2

Correct

Mark 1.00 out of 1.00

Given that

$$2(\mathbf{a} - \mathbf{b}) + 3(\mathbf{x} - \mathbf{a}) = 2\mathbf{b},$$

express $\mathbf{x}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(Type `a` to enter $\mathbf{a}$ and to enter `b` type $\mathbf{b}$.)

$\mathbf{x} =$ (a+4b)/3

Your answer is correct.

Since

$$2(\mathbf{a} - \mathbf{b}) + 3(\mathbf{x} - \mathbf{a}) = 2\mathbf{b},$$

expanding the brackets gives

$$2\mathbf{a} - 2\mathbf{b} + 3\mathbf{x} - 3\mathbf{a} = 2\mathbf{b},$$

so

$$\begin{aligned} 3\mathbf{x} &= 2\mathbf{b} - 2\mathbf{a} + 2\mathbf{b} + 3\mathbf{a} \\ &= \mathbf{a} + 4\mathbf{b} \end{aligned}$$

and hence

$$\mathbf{x} = \frac{1}{3}(\mathbf{a} + 4\mathbf{b}).$$

See Unit 5, Subsection 5.2.

Questions

1 2 3

Question 3

Correct

Mark 1.00 out of 1.00

A ship sails on a course with bearing 60° for 2 miles, then turns and sails 3 miles on a bearing of 310°.

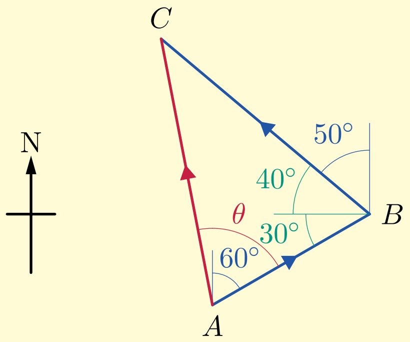
What is the distance (in miles, to one decimal place) and the bearing (to the nearest degree) of its final location from its initial location?

The distance is 3.0 miles (to 1 d.p.).

The bearing is 349 ° (to the nearest degree).

Your answer is correct.

Let A be the initial position of the ship, B the point at which it turns and C the final position. The situation is illustrated below.



At A, the angle between $\overrightarrow{AB}$ and East is 30°, so at B, the angle between $\overrightarrow{AB}$ and West is 30°.

At B, the angle between $\overrightarrow{BC}$ and West is 40°. So the angle at B within the triangle ABC is 70°.

We know that $|\overrightarrow{AB}| = 2$ and $|\overrightarrow{BC}| = 3$.

By the cosine rule in triangle ABC,

$$\begin{aligned}|\overrightarrow{AC}|^2 &= |\overrightarrow{AB}|^2 + |\overrightarrow{BC}|^2 - 2|\overrightarrow{AB}||\overrightarrow{BC}|\cos 70^\circ \\ &= 4 + 9 - 2 \times 2 \times 3 \cos 70^\circ \\ &= 8.8957\ldots\end{aligned}$$

and hence $|\overrightarrow{AC}| = 2.9825\ldots$. To one decimal place, $|\overrightarrow{AC}| = 3.0$.

By the sine rule in triangle ABC, $\dfrac{|\overrightarrow{BC}|}{\sin\theta} = \dfrac{|\overrightarrow{AC}|}{\sin 70^\circ}$,

that is, $\dfrac{3}{\sin\theta} = \dfrac{2.9825\ldots}{\sin 70^\circ}$.

So $\sin\theta = \dfrac{3 \times \sin 70^\circ}{2.9825\ldots} = 0.945\ldots$

and hence $\theta = 71^\circ$ to the nearest degree.

So the angle between $\overrightarrow{AC}$ and North is $71^\circ - 60^\circ = 11^\circ$, and so the bearing of C from A is $360^\circ - 11^\circ = 349^\circ$, both to the nearest degree.

The final position of the ship is 3.0 miles (to 1 d.p.) from the initial position on a bearing of 349° (to the nearest degree).

See Unit 5, Subsection 5.3.