The Open University

Faculty of Science, Technology, Engineering and Mathematics

MST210
DIAGNOSTIC QUIZ

Am I ready to start on MST210?

The diagnostic quiz below is designed to help you to answer this question. This document also contains some advice on preparatory work that you may find useful before starting on MST210 (see below and on page 12).

MST210 is designed to folllow on from study of MST124 and MST125. These modules should be an excellent preparation for MST210. If you have studied these modules then you should complete this diagnostic quiz to identify any parts of these modules that you may need to brush up on before starting MST210.

If you have studied MST121 and MS221 then in addition to finding out topics you may need to brush up on this diagnostic quiz will help you decide on which topics from the MST210 bridging material that you need to study. The MST210 bridging material contains those topics relevant to MST210 that are included in MST124 and MST125 that are not included in MST121 and MS221.

The mathematical prerequisites required for MST210 can be separated into categories:

A prerequisites that that are taught in MST121 and MS221;

B prerequisites that that are taught in MST124 and MST125, that are not taught in MST121 and MS221;

The diagnostic quiz below is divided into sections A and B corresponding to these two categories.

You should be confident about approaching all questions in this diagnostic quiz before starting MST210. If you are happy with section A, but not happy (or are unfamiliar with) topics in section B then you should study the MST210 bridging material before starting MST210.

Do contact your Student Support Team via StudentHome if you have any queries about your suitability for the module.

Diagnostic Quiz – Questions

Section A

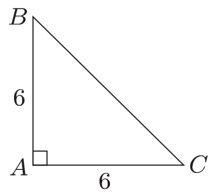
1 Using a calculator, give the values of each of the following to three decimal places:

(a) $\tan(1.2)$ (where 1.2 is in radians);

(b) $e^{-2.731}$;

(c) $\ln(4/27)$.

2 In the triangle BAC, the angle BAC is a right angle, and the sides AB and AC are each of length 6.

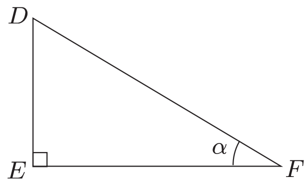


(a) Give each of the angles in triangle ABC in degrees and in radians.

(b) What is the length BC?

(c) What is the area of triangle ABC?

3 (a) In the triangle DEF, the angle DEF is a right angle, and angle EFD is α. Write down each of $\cos\alpha$, $\sin\alpha$ and $\tan\alpha$ as ratios of sides in the triangle DEF.



(b) Give the values of $\cos(180°)$ and $\sin(270°)$.

4 Solve for x each of the following equations.

(a) $3x + 4 = 10$

(b) $3(x + 3) - 7(x - 1) = 0$

(c) $\dfrac{2}{1+x} = \dfrac{3}{2-x}$

(d) $\sqrt{x^2 + 7} = 4$

5 (a) Make t the subject of the equation

$$x = x_0 - \tfrac{1}{2}gt^2.$$

(b) Make x the subject of the equation

$$\sqrt{\frac{x-2}{x+3}} = t.$$

6 Give the equation of the straight line passing through the points $y = 2$ when $x = 0$ and $y = 8$ when $x = 2$. What is the gradient of this line?

7 If $y(x) = 3 + 2x - \sin(2x)$, what is $y\left(\frac{\pi}{2}\right)$?

8 (a) Solve for y the equation

$$2y^2 - 4y + 1 = 0.$$

(b) Solve for λ the equation

$$\lambda^2 + 4\lambda + 4 = 0.$$

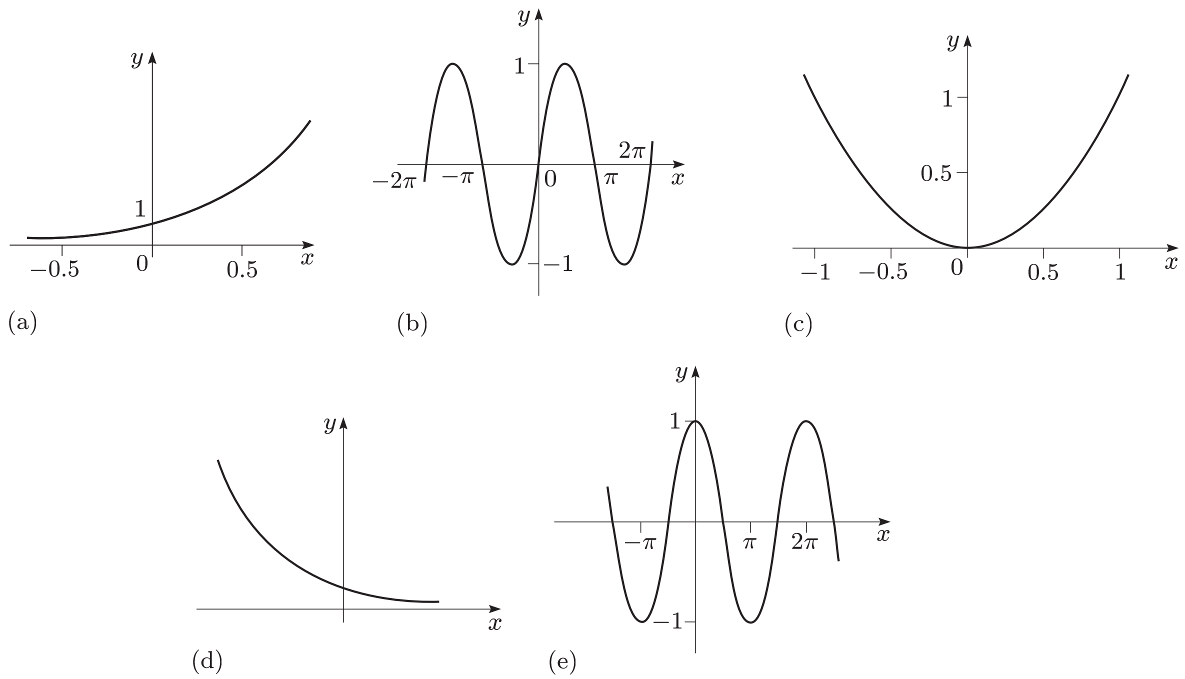
9 Solve the following simultaneous equations for x and y:

$$2x - y = 3,$$
$$3x + y = 2.$$

10 You plan to make some cakes and some biscuits for a charity sale. Each cake uses 2 eggs and 0.11 kg of butter. Each batch of biscuits requires 1 egg and 0.15 kg of butter. You have only 12 eggs and 1 kg of butter, and you want to use all of these (so far as is possible). (Each recipe also requires other ingredients, but you have plenty of those.) How many cakes and how many batches of biscuits should you make?

11 Five graphs are given in parts (a)–(e) of the figure below. Each graph is that of one of the functions (i)–(v). Match each graph with the appropriate function.

Functions: (i) $y(x) = e^{-x}$, (ii) $y(x) = e^{2x}$, (iii) $y(x) = \sin x$, (iv) $y(x) = \cos x$, (v) $y(x) = x^2$.



12 Simplify each of the following.

(a) x^2x^5 (b) x^3/x^4 (c) $(x^2)^3$ (d) $9^{1/2}$

13 Express $(e^{-2x} \times e^{3x})^2$ in the form e^y.

14 Express $\frac{1}{2}\ln(25) + 3\ln(\frac{1}{2})$ in the form $\ln(y)$.

15 Show that $y = \ln(2e^{-x/2})$ is the equation of a straight line. What is the gradient of this line?

16 Solve for y the equation

$$\ln(y) = 2\ln(x) - 1.$$

17 If $|x - 2| < 10^{-2}$, what range of values can x take?

18 Solve for ν the equation below (where $m \neq 0$):

$$-\frac{m}{gr} = -\frac{\mu m}{\nu^2}.$$

19 (a) What solutions for x has the equation $\sin x = 1$?

(b) What value does your calculator give for $\arcsin(1)$?

20 What is $\cos^2\alpha + \sin^2\alpha$ (where α may be any real number)?

21 Use the trigonometric identity $\cos(a + b) = \cos a \cos b - \sin a \sin b$, and particular values of a and b, to simplify $\cos(\pi + x)$.

22 Find the local maxima and minima of the function

$$y(x) = 2x^3 - 3x^2 - 12x + 6.$$

23 (a) Find $\dfrac{ds}{dt}$ where $s = 5e^{3t}$.

(b) Find $y'(t)$ where $y(t) = 3t^5 - 10\sqrt{t}$.

(c) Find $\dfrac{dz}{dx}$ where $z = 14\sin(x/8)$.

24 Evaluate each of the following integrals.

(a) $\displaystyle\int (1 + 6x^3)\,dx$ (b) $\displaystyle\int_0^\pi \sin(3t)\,dt$

25 Suppose that

$$-\frac{m}{gr} \geq -\frac{\mu m}{\nu^2},$$

where m, r, g and μ are positive.

(a) Rearrange this inequality by multiplying each side first by $-\nu^2$, then by gr/m.

(b) In terms of the other parameters, what is the largest value that ν can take?

26 (a) (i) Find $y'(t)$ where $y(t) = t\sin(3t)$.

(ii) Find $\dfrac{dx}{dt}$ where $x = \ln(t^3 + 1)$.

(b) Find the velocity at time $t = 3$ of an object whose position at time t is given by $x(t) = e^{-2t}\cos(\frac{\pi}{3}t)$.

27 (a) Use integration by substitution to find $\displaystyle\int x^2 \exp(2 + 3x^3)\,dx$.

(b) Use integration by parts to find $\displaystyle\int x \ln x\,dx$.

28 Express the complex number $(1 + i)(3 + 2i)$ in the form $a + bi$.

29 Find the imaginary part of the complex number $(e^{2+i\pi})^3$.

Section B

30 Use the integrating factor method to find the general solution of the following differential equation.

$$\frac{dy}{dx} - y = e^x \sin x$$

31 Use the integrating factor method to solve the following initial-value problem.

$$t\dot{y} + 2y = t^2, \quad y(1) = 1$$

32 During December, a large plastic Christmas tree of mass 10 kg is suspended by its apex using two ropes attached to buildings either side of the high street of Trappendorf. The ropes make angles of $\frac{\pi}{6}$ and $\frac{\pi}{4}$ with the horizontal. Model the Christmas tree as a particle and the ropes as light, inextensible strings. What are the magnitudes of the tension forces due to the two ropes?

33 A particle is moving in a straight line along the x-axis. At time t the particle has an acceleration given by

$$\mathbf{a}(t) = (18t - 20)\mathbf{i} \quad (t \geq 0).$$

Initially, at $t = 0$, the particle has position $\mathbf{r}(0) = 7\mathbf{i}$ and velocity $\mathbf{v}(0) = 3\mathbf{i}$. Find the position of the particle at time $t = 10$.

34 A stone, dropped from rest, takes 3 seconds to reach the bottom of a well. Assuming that the only force acting on the stone is gravity, estimate:

(a) the depth of the well;

(b) the speed of the stone when it reaches the bottom.

Diagnostic Quiz – Answers

Section A

1 (a) 2.572

(b) 0.065

(c) −1.910

2 (a) $\angle ABC = \angle ACB = 45^\circ = \frac{\pi}{4}$ radians.

$\angle BAC = 90^\circ = \frac{\pi}{2}$ radians.

(b) By Pythagoras's Theorem,

$$BC^2 = AB^2 + AC^2 = 6^2 + 6^2 = 72,$$

so

$$BC = \sqrt{72} = 6\sqrt{2}.$$

(c) The area of a triangle equals half its base times its height, so the area of triangle ABC is

$\frac{1}{2} \times 6 \times 6 = 18$ square units.

3 (a) $\cos\alpha = \dfrac{EF}{DF}$, $\sin\alpha = \dfrac{DE}{DF}$, $\tan\alpha = \dfrac{DE}{EF}$.

(b) $\cos(180^\circ) = -1$, $\sin(270^\circ) = -1$.

4 (a) $3x + 4 = 10$

$3x = 6$

$x = 2$

(b) $3(x+3) - 7(x-1) = 0$

$3x + 9 - 7x + 7 = 0$

$-4x + 16 = 0$

$x = 4$

(c) $\dfrac{2}{1+x} = \dfrac{3}{2-x}$

$2(2-x) = 3(1+x)$

$4 - 2x = 3 + 3x$

$5x = 1$

$x = \frac{1}{5}$

(d) $\sqrt{x^2+7} = 4$

$x^2 + 7 = 4^2 = 16$

$x^2 = 9$

$x = \pm\sqrt{9}$

So $x = 3$ or $x = -3$.

5 (a) $x = x_0 - \frac{1}{2}gt^2$

$gt^2 = 2(x_0 - x)$

$t = \pm\sqrt{\dfrac{2}{g}(x_0 - x)}$

(b) $\sqrt{\dfrac{x-2}{x+3}} = t$

$\dfrac{x-2}{x+3} = t^2$

$x - 2 = t^2(x+3) = t^2x + 3t^2$

$x(1-t^2) = 2 + 3t^2$

$x = \dfrac{2+3t^2}{1-t^2}$

6 The equation of a straight line has the form

$y = mx + c.$

To satisfy the given conditions, the constants m and c must satisfy the equations

$2 = c$ (since $y = 2$ when $x = 0$),

$8 = 2m + c$ (since $y = 8$ when $x = 2$).

Thus $c = 2$ and $m = 3$, so the required equation is

$y = 3x + 2.$

The gradient of this line is given by m, and so is 3.

7

$$\begin{aligned} y\left(\tfrac{\pi}{2}\right) &= 3 + 2 \times \tfrac{\pi}{2} - \sin\left(2 \times \tfrac{\pi}{2}\right) \\ &= 3 + \pi - \sin\pi \\ &= 3 + \pi - 0 \\ &= 3 + \pi \end{aligned}$$

8 Use the formula for solving a quadratic equation.

(a)
$$\begin{aligned} y &= \frac{4 \pm \sqrt{(-4)^2 - 4(2)(1)}}{2(2)} \\ &= \frac{4 \pm \sqrt{8}}{4} = 1 \pm \tfrac{1}{2}\sqrt{2} \end{aligned}$$

(b) $\lambda = \dfrac{-4 \pm \sqrt{16-16}}{2} = -2$

(The expression $\lambda^2 + 4\lambda + 4$ is a perfect square, $(\lambda+2)^2$.)

[*Unit 1*, Subsection 2.3]

9 Adding the equations gives $5x = 5$, so $x = 1$. Then the first equation gives $2 - y = 3$, so $y = -1$.

The solution is $x = 1$, $y = -1$.

[*Unit 1*, Subsection 2.2]

10 Suppose that you make C cakes and B batches of biscuits. To use all the ingredients, you would need

$$2C + B = 12 \quad \text{(for the eggs)},$$
$$C(0.11) + B(0.15) = 1 \quad \text{(for the butter)}.$$

The first equation gives $B = 12 - 2C$, and then the second equation gives

$$C(0.11) + (12 - 2C)(0.15) = 1.$$

Solving this equation for C gives $C = 4.2$ (to one decimal place), and then $B = 3.6$.

You might be able to make the equivalent of 4.2 cakes (by making 4 cakes each just a little bit larger than the recipe). But this would imply using 8.4 eggs on the cakes, which does not look very feasible (though you could try to divide an egg).

The best bet would seem to be to make 4 cakes and 4 batches of biscuits. This will use up all 12 eggs. It really needs 1.04 kg of butter – a bit more than you have. Either steal a scrap from your butter dish, or, if that's not possible, skimp a bit on the butter for the biscuits. (Scale down the other ingredients very slightly if you're worried, so that you're being slightly generous on the eggs rather than mean on the butter.)

The main point here is that, when you are applying mathematics, you may need to use your common sense to relate the results of a mathematical calculation to the real problem.

11 The matching is as follows.

(a)(ii) (b)(iii) (c)(v) (d)(i) (e)(iv)

[*Unit 1*, Sections 2 and 3]

12 We use the rules for manipulating indices.

(a) $x^2x^5 = x^{2+5} = x^7$

(b) $x^3/x^4 = x^{3-4} = x^{-1} (= 1/x)$

(c) $(x^2)^3 = x^{2\times 3} = x^6$

(d) $9^{1/2} = \sqrt{9} = 3$

[*Unit 1*, Subsection 2.4]

13 $(e^{-2x} \times e^{3x})^2 = (e^{3x-2x})^2 = (e^x)^2 = e^{2x}$

[*Unit 1*, Subsection 2.4]

14
$$\begin{aligned}\tfrac{1}{2}\ln(25) + 3\ln(\tfrac{1}{2}) &= \ln(\sqrt{25}\,) + \ln((\tfrac{1}{2})^3) \\ &= \ln(5) + \ln(\tfrac{1}{8}) \\ &= \ln(\tfrac{5}{8})\end{aligned}$$

[*Unit 1*, Subsection 2.4]

15 Using the properties of exponentials and logarithms,

$$\ln(2e^{-x/2}) = \ln(2) + \ln(e^{-x/2}) = \ln(2) - \tfrac{1}{2}x,$$

so

$$y = \ln(2) - \tfrac{1}{2}x.$$

This is the equation of a straight line.

The gradient of the straight line is $-\frac{1}{2}$ (the coefficient of x).

16 If

$$\ln(y) = 2\ln(x) - 1,$$

then taking exponentials of each side gives

$$\begin{aligned} y = \exp(\ln(y)) &= \exp(2\ln(x) - 1) \\ &= \exp(\ln(x^2) - 1) \\ &= \exp(\ln(x^2))/\exp(1) \\ &= x^2/e. \end{aligned}$$

17 Recall that $|y|$ means y if $y \geq 0$, and $-y$ if $y < 0$. If

$$|x - 2| < 10^{-2},$$

then

$$-10^{-2} < x - 2 < 10^{-2},$$

so

$$2 - 10^{-2} < x < 2 + 10^{-2},$$

i.e.

$$1.99 < x < 2.01.$$

18 If

$$-\frac{m}{gr} = -\frac{\mu m}{\nu^2},$$

then multiplying each side by $-\dfrac{\nu^2}{m}$ gives

$$\left(-\frac{\nu^2}{m}\right)\left(-\frac{m}{gr}\right) = \left(-\frac{\nu^2}{m}\right)\left(-\frac{\mu m}{\nu^2}\right),$$

i.e.

$$\frac{\nu^2}{gr} = \mu.$$

Hence

$$\nu = \pm\sqrt{\mu g r}.$$

19 (a) $\sin x = 1$ when $x = \frac{\pi}{2}$, or when x differs from $\frac{\pi}{2}$ by a multiple of 2π.

(b) My calculator gives $\arcsin(1) = 90$, but that is because it is working in degrees. If your calculator is working in radians (as it will need to be for MST209), then it should give

$$\arcsin(1) = 1.570796327 \quad (\text{i.e. } \tfrac{\pi}{2}).$$

20 $\cos^2\alpha + \sin^2\alpha = 1.$

21 We have

$$\begin{aligned}\cos(\pi + x) &= \cos\pi\cos x - \sin\pi\sin x\\ &= (-1)\cos x - (0)\sin x\\ &= -\cos x.\end{aligned}$$

22 To find local maxima and minima, first find the stationary points, where $\frac{dy}{dx} = 0$.

Differentiating $y = 2x^3 - 3x^2 - 12x + 6$ gives

$$\frac{dy}{dx} = 6x^2 - 6x - 12.$$

So to find the stationary points, solve

$$6x^2 - 6x - 12 = 0,$$

i.e.

$$x^2 - x - 2 = 0.$$

To solve this quadratic equation, you can either use the formula, or factorize to obtain

$$(x - 2)(x + 1) = 0.$$

Thus there are stationary points at $x = 2$ and $x = -1$.

Now $\frac{d^2y}{dx^2} = 12x - 6$.

At $x = -1$, this is negative, so there is a local maximum at $x = -1$, of value $y = 13$.

At $x = 2$, this second derivative is positive, so there is a local minimum at $x = 2$, of value $y = -14$.

23 (a) If $s = 5e^{3t}$, then

$$\frac{ds}{dt} = 5(3e^{3t}) = 15e^{3t}.$$

(b) If $y(t) = 3t^5 - 10\sqrt{t}$, then

$$y'(t) = 15t^4 - 5t^{-1/2}.$$

(c) If $z = 14\sin(x/8)$, then

$$\frac{dz}{dx} = \tfrac{14}{8}\cos(x/8) = \tfrac{7}{4}\cos(x/8).$$

24 (a) This is an indefinite integral:

$$\begin{aligned}\int (1 + 6x^3)\,dx &= x + \tfrac{6}{4}x^4 + c\\ &= x + \tfrac{3}{2}x^4 + c,\end{aligned}$$

where c is an arbitrary constant.

(b) This is a definite integral:

$$\begin{aligned}\int_0^\pi \sin(3t)\,dt &= \left[-\tfrac{1}{3}\cos(3t)\right]_0^\pi\\ &= -\tfrac{1}{3}(\cos(3\pi) - \cos(0))\\ &= -\tfrac{1}{3}(-1 - 1) = \tfrac{2}{3}.\end{aligned}$$

25 We have

$$-\frac{m}{gr} \geq -\frac{\mu m}{\nu^2}.$$

(a) The quantity $-\nu^2$ is negative, so on multiplying both sides by $-\nu^2$, we must reverse the inequality:

$$\frac{m}{gr}\nu^2 \leq \mu m.$$

Then (since gr/m is positive)

$$\nu^2 \leq \mu gr.$$

(b) The largest value that ν can take is $\sqrt{\mu gr}$.

26 (a) (i) To differentiate $y(t) = t\sin(3t)$, use the product rule. We obtain

$$y'(t) = \sin(3t) + 3t\cos(3t).$$

(ii) To differentiate $x = \ln(t^3 + 1)$, use the composite (or 'function of a function') rule. We obtain

$$\frac{dx}{dt} = 3t^2 \times \frac{1}{t^3 + 1} = \frac{3t^2}{t^3 + 1}.$$

(b) To find the velocity $v(t)$ of the object, we differentiate the expression for its position, i.e.

$$\begin{aligned}v(t) &= \frac{dx}{dt}\\ &= -2e^{-2t}\cos(\tfrac{\pi}{3}t) - \tfrac{\pi}{3}e^{-2t}\sin(\tfrac{\pi}{3}t)\\ &= -e^{-2t}(2\cos(\tfrac{\pi}{3}t) + \tfrac{\pi}{3}\sin(\tfrac{\pi}{3}t)).\end{aligned}$$

So at time $t = 3$, the object's velocity is

$$\begin{aligned}v(3) &= -e^{-6}(2\cos\pi - \tfrac{\pi}{3}\sin\pi)\\ &= -e^{-6}(2(-1) + \tfrac{\pi}{3}(0))\\ &= 2e^{-6}\\ &\simeq 0.005.\end{aligned}$$

27 (a) Integration by substitution uses the formula

$$\int f(u)\frac{du}{dx}\,dx = \int f(u)\,du.$$

With $u(x) = 2 + 3x^3$, we have $\frac{du}{dx} = 9x^2$, and then

$$\begin{aligned}\int x^2\exp(2 + 3x^3)\,dx &= \int \tfrac{1}{9}\frac{du}{dx}\exp u\,dx\\ &= \tfrac{1}{9}\int \exp u\,\frac{du}{dx}\,dx\\ &= \tfrac{1}{9}\int \exp u\,du\\ &= \tfrac{1}{9}\exp u + c\\ &= \tfrac{1}{9}\exp(2 + 3x^3) + c,\end{aligned}$$

where c is an arbitrary constant.

(b) Integration by parts uses the formula

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx.$$

With $f(x) = \ln x$ and $g(x) = \frac{1}{2}x^2$, we have $f'(x) = 1/x$ and $g'(x) = x$, and then

$$\int x\ln x\,dx = \tfrac{1}{2}x^2\ln x - \int \tfrac{1}{2}x\,dx$$
$$= \tfrac{1}{2}x^2\ln x - \tfrac{1}{4}x^2 + c,$$

where c is an arbitrary constant.

28 Since $i^2 = -1$, we have

$$(1+i)(3+2i) = 3+2i+3i+2i^2$$
$$= 3+5i-2$$
$$= 1+5i.$$

29 We have

$$(e^{2+i\pi})^3 = e^{(2+i\pi)\times 3}$$
$$= e^{6+3i\pi}$$
$$= e^6(\cos(3\pi) + i\sin(3\pi)).$$

The imaginary part of this is

$$e^6\sin(3\pi) = 0.$$

Section B

30 The given equation is $dy/dx - y = e^x\sin x$. The integrating factor for this differential equation is

$$p = \exp\left(\int(-1)\,dx\right) = \exp(-x) = e^{-x}.$$

Multiplying through by $p(x)$ gives

$$e^{-x}\frac{dy}{dx} - e^{-x}y = \sin x.$$

Thus the differential equation can be rewritten as

$$\frac{d}{dx}(e^{-x}y) = \sin x.$$

On integrating, we find the general solution

$$e^{-x}y = -\cos x + C,$$

or, equivalently,

$$y = e^x(C - \cos x),$$

where C is an arbitrary constant.

31 After division by t, the given equation can be written as $dy/dt + (2/t)y = t$, which is the standard form of equation for the integrating factor method. (To avoid division by zero, we take $t > 0$, say, which is consistent with the initial condition.)

The integrating factor is

$$p = \exp\left(\int \frac{2}{t}\,dt\right)$$
$$= \exp(2\ln t)$$
$$= \exp(\ln(t^2))$$
$$= t^2.$$

Multiplying through by $p(t)$ gives

$$t^2\frac{dy}{dt} + 2ty = t^3.$$

Thus the differential equation can be rewritten as

$$\frac{d}{dt}(t^2y) = t^3.$$

On integrating, we find the general solution

$$t^2y = \tfrac{1}{4}t^4 + C, \quad \text{or, equivalently,} \quad y = \tfrac{1}{4}t^2 + Ct^{-2},$$

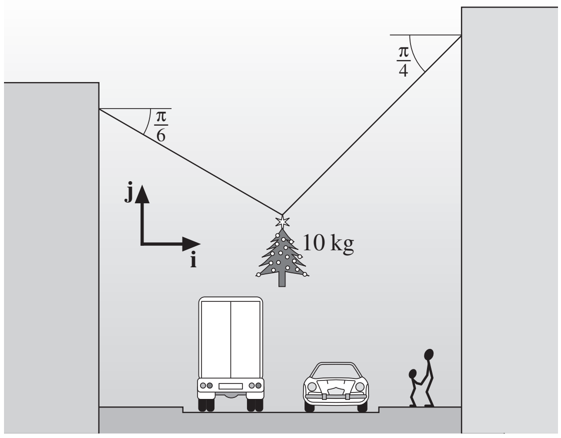
where C is an arbitrary constant.

From the initial condition $y(1) = 1$, we have $1 = \frac{1}{4} + C$, so $C = \frac{3}{4}$. Hence the solution of the initial-value problem is

$$y = \tfrac{1}{4}(t^2 + 3t^{-2}).$$

32 In this and other solutions, you may find that your diagrams and chosen axes are different from those given. You should still be able to check the validity of your solution against the given one, as the basic concepts are unchanged by these differences. Any choice of axes should lead to the same final answers as those given.

◀Draw picture▶

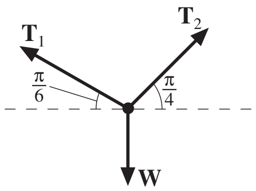


◀Choose axes▶

The forces all lie in a vertical plane, so we need only two axes, as shown in the diagram above.

◀Draw force diagram▶

The force diagram, where the tension forces are denoted by $\mathbf{T}_1$ and $\mathbf{T}_2$, and the weight of the tree by $\mathbf{W}$, is as follows.



◀Apply law(s)▶

The equilibrium condition for particles gives

$$\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{W} = \mathbf{0}. \quad \text{(S.1)}$$

◀Solve equation(s)▶

From the above force diagram we have

$$\mathbf{W} = -|\mathbf{W}|\,\mathbf{j} = -10g\mathbf{j}.$$

The other forces can be expressed in terms of components:

$$\begin{aligned}\mathbf{T}_1 &= (\mathbf{T}_1 \cdot \mathbf{i})\,\mathbf{i} + (\mathbf{T}_1 \cdot \mathbf{j})\,\mathbf{j}\\ &= |\mathbf{T}_1| \cos\tfrac{5\pi}{6}\,\mathbf{i} + |\mathbf{T}_1| \cos\tfrac{\pi}{3}\,\mathbf{j}\\ &= -\tfrac{\sqrt{3}}{2}|\mathbf{T}_1|\,\mathbf{i} + \tfrac{1}{2}|\mathbf{T}_1|\,\mathbf{j},\end{aligned}$$

$$\begin{aligned}\mathbf{T}_2 &= (\mathbf{T}_2 \cdot \mathbf{i})\,\mathbf{i} + (\mathbf{T}_2 \cdot \mathbf{j})\,\mathbf{j}\\ &= |\mathbf{T}_2| \cos\tfrac{\pi}{4}\,\mathbf{i} + |\mathbf{T}_2| \cos\tfrac{\pi}{4}\,\mathbf{j}\\ &= \tfrac{1}{\sqrt{2}}|\mathbf{T}_2|\,\mathbf{i} + \tfrac{1}{\sqrt{2}}|\mathbf{T}_2|\,\mathbf{j}.\end{aligned}$$

Resolving (S.1) in the **i**-direction gives

$$-\tfrac{\sqrt{3}}{2}|\mathbf{T}_1| + \tfrac{1}{\sqrt{2}}|\mathbf{T}_2| + 0 = 0. \quad \text{(S.2)}$$

Similarly, resolving (S.1) in the **j**-direction gives

$$\tfrac{1}{2}|\mathbf{T}_1| + \tfrac{1}{\sqrt{2}}|\mathbf{T}_2| - 10g = 0. \quad \text{(S.3)}$$

Subtracting (S.2) from (S.3) gives

$$(\tfrac{1}{2} + \tfrac{\sqrt{3}}{2})|\mathbf{T}_1| - 10g = 0,$$

so

$$|\mathbf{T}_1| = \tfrac{20}{1+\sqrt{3}}g \simeq 71.81.$$

Substituting this value of $|\mathbf{T}_1|$ into (S.2) gives

$$|\mathbf{T}_2| = \tfrac{20\sqrt{3}}{\sqrt{2}+\sqrt{6}}g \simeq 87.95.$$

◀Interpret solution▶

The model predicts that the magnitudes of the tension forces due to the ropes are about 72 N and 88 N.

33 Since

$$a(t) = \frac{dv}{dt} = 18t - 20,$$

we have

$$v = \int (18t - 20)\,dt = 9t^2 - 20t + A.$$

Using the initial condition $v(0) = 3$, we obtain $A = 3$. Hence the velocity of the particle is given by

$$v(t) = 9t^2 - 20t + 3.$$

Now $v(t) = dx/dt$, so

$$x = \int (9t^2 - 20t + 3)\,dt = 3t^3 - 10t^2 + 3t + B.$$

The initial condition $x(0) = 7$ gives $B = 7$. Hence the position of the particle is given by

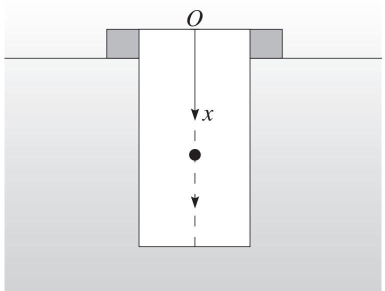
$$x(t) = 3t^3 - 10t^2 + 3t + 7.$$

Substituting $t = 10$ into this expression gives

$$x(10) = 3000 - 1000 + 30 + 7 = 2037,$$

so at time $t = 10$ the particle is 2037 units along the positive x-axis.

34 ◀Draw picture▶



◀Choose axes▶

The x-axis is chosen to point vertically downwards, with the origin at the top of the well, as shown above.

◀Draw force diagram▶

The model assumes that the only force is gravity, so the force diagram is as follows.

◀Apply Newton's 2nd law▶

Applying Newton's second law to the stone gives $\mathbf{W} = m\mathbf{a}$. Since $\mathbf{W} = mg\mathbf{i}$, we have $m\mathbf{a} = mg\mathbf{i}$, and resolving in the **i**-direction gives

$$a = g.$$

◀Solve differential equation▶

Using $a = dv/dt$, we obtain

$$\frac{dv}{dt} = g.$$

Integrating this gives

$$v = gt + A.$$

The initial condition that the stone is dropped from rest ($v = 0$ when $t = 0$) gives $A = 0$. Hence

$$v = gt. \tag{S.4}$$

Now using $v = dx/dt$, we have

$$\frac{dx}{dt} = gt.$$

Integrating this gives

$$x = \tfrac{1}{2}gt^2 + B.$$

The initial condition $x = 0$ when $t = 0$ gives $B = 0$. So

$$x = \tfrac{1}{2}gt^2. \tag{S.5}$$

◀Interpret solution▶

(a) Using Equation (S.5) with $t = 3$ gives

$$x = \tfrac{1}{2} \times 9.81 \times 3^2 = 44.15.$$

So the well is estimated to be about 44 m deep.

(b) Using Equation (S.4) with $t = 3$ gives

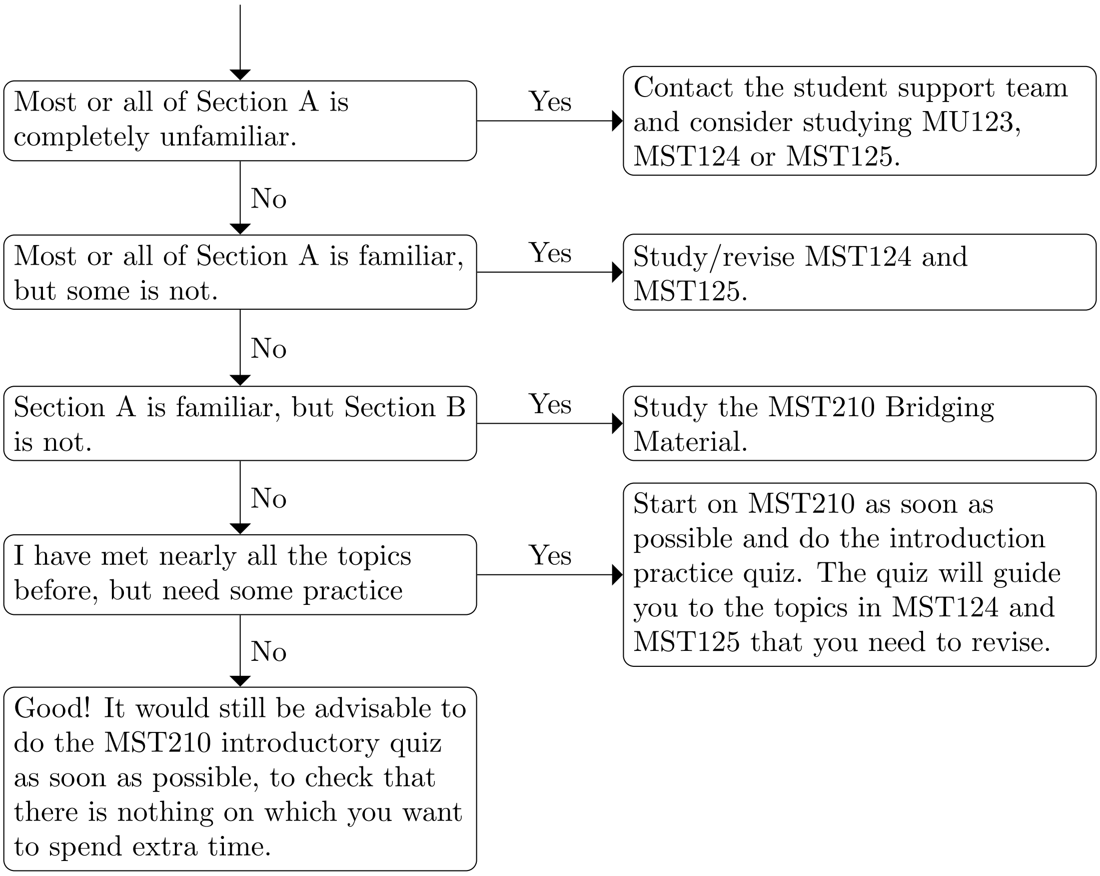
$$v = 9.81 \times 3 = 29.43.$$

So the predicted speed of the stone as it reaches the bottom is about $29\,\mathrm{m\,s^{-1}}$.

What can I do to prepare for MST210?

After completing the diagnostic quiz use the following flowchart to decide whether MST210 is an appropriate module for you, and what you should do by way of preparation before the module starts.

Do contact your Student Support Team via StudentHome if you have any queries about your suitability for the module.



Materials that you can use to prepare for MST210

MST210 Bridging Material

This introduces ideas used in MST209 that are covered in MST124 and MST125 but not in MST121 and MS221. This contains topics such as the integrating factor method for solving first-order differential equations; the cross product of two vectors; and an introduction to Newtonian mechanics.

MST210 introduction practice quiz

MST210 starts with an online diagnostic quiz to help you determine what topics you need to revise from MST124 and MST125. The quiz will guide you to the relevant sections of the MST124 and MST125 units, copies of which are included on the module website.

