Are you ready for MST124?

Solutions

Please ensure you have completed all the questions in the *Are you ready for MST124?* quiz to the best of your ability before opening this document.

Mark your answers according to the correct answers given here. There is one mark for each question.

Study advice based on your score is given on the following pages.

Preparatory study

If you wish to do some further preparatory study before starting *Essential mathematics 1* (MST124), then try working through a GCSE Higher Level mathematics, or equivalent, text book, which may be available online or in a local library. If you have difficulties with algebra, you should consider signing-up for *Discovering mathematics* (MU123).

Have you previously studied MU123 and/or an A- or AS-level in mathematics (or a Higher in Scotland), a relevant HNC/D or an International Baccalaureate® Diploma Programme? If so, you may like to review your work for that in preparation for MST124.

You may also like to look at the sections of the following external websites which relate to the topics in the MST124 quiz, such as algebra and trigonometry:

- Mathtutor: `http://www.mathtutor.ac.uk/`
- Khan academy maths videos: `https://www.khanacademy.org/`
- Firefly lectures: `http://fireflylectures.com/`

You could also ensure that you are thoroughly familiar with how your scientific calculator works.

Once you've signed-up, there will be some material available to help you brush up on your existing skills. However, if you are unlikely to have time to do preparatory work before starting your module, you should consider doing MU123 first.

If you would like to discuss whether you are ready for *Essential mathematics 1* (MST124), please contact Student Recruitment (tel: +44 (0) 300 303 5303, email: general-enquiries@open.ac.uk), who, if necessary, will pass you on to an educational adviser. It would be helpful to give them your score on the quiz and how long it took you to complete it.

If you are already registered as an Open University student you can make an early start on MST124 by looking at the first two units at `learn1.open.ac.uk/mod/subpage/view.php?id=4448`

Study advice

If your score is 15 or more out of 20, then congratulations, you've achieved an excellent result on this quiz.

Your score suggests that you should be ready to proceed with *Essential mathematics 1* (MST124). However, the more confident and fluent you are with key topics such as algebra, trigonometry and geometry, and how the main functions (including the sine and exponential keys) on a scientific calculator work, the easier you'll find it to study *Essential mathematics 1* (MST124). It would do no harm to brush up on any topics you feel less confident with, and to keep practising your skills in the key topics.

If your score is between 10 and 14 out of 20, then well done, you've achieved a good result on this quiz.

Your score suggests that as long as you can find time to brush up on topics for which you didn't achieve full marks then you are probably ready to proceed with *Essential mathematics 1* (MST124). The content of MST124 assumes you already have good skills in algebra, trigonometry and geometry, and familiarity with the main functions on a scientific calculator, such as the sine and exponential functions, so take the time to practice these as much as possible.

If your score is between 8 and 13 out of 20, then you've achieved good results in some of the questions on this quiz.

However, your overall score suggests that you might find *Essential mathematics 1* (MST124) quite a challenge. It assumes you already have fluent skills in algebra, trigonometry and geometry, and familiarity with the main functions on a scientific calculator, such as the sine and exponential functions.

Instead you should consider starting with *Discovering mathematics* (MU123). For further advice about choosing between MU123 and MST124, visit the MathsChoices website: `mathschoices.open.ac.uk`.

If your choice of qualification doesn't offer a route which includes *Discovering mathematics* (MU123), then we recommend you contact our Student Registration and Enquiry Service. You will also need to allow time to improve your skills in the key topics required for *Essential mathematics 1* (MST124).

If you have plenty of time in which to revise and practice before the start date of *Essential mathematics 1* (MST124) then follow the advice given on the previous page to improve your understanding of the key topics.

If your score is 7 or less out of 20, then you should consider starting with *Discovering mathematics* (MU123). For further advice about choosing between MU123 and MST124, visit the MathsChoices website: `mathschoices.open.ac.uk`.

If your choice of qualification doesn't offer a pathway which includes *Discovering mathematics* (MU123), then you should contact our Student Registration and Enquiry Service.

Solutions

Question 1

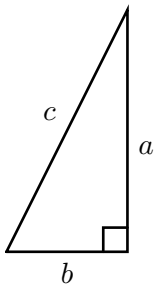
Substitute the values $a = -4$ and $b = 3$ into the expression to give

$$\begin{aligned} a(a-b) - ab - a^2 &= (-4) \times ((-4) - 3) - (-4) \times 3 - (-4)^2 \\ &= (-4) \times (-7) - (-12) - 16 \\ &= 28 + 12 - 16 \\ &= 24. \end{aligned}$$

The answer is 24. 1 mark

Question 2

The ladder, the floor and the wall form a right-angled triangle, as shown on the right. Pythagoras' theorem says that the sides lengths a, b and c satisfy $c^2 = a^2 + b^2$, so that $a^2 = c^2 - b^2$.



You were told that $c = 7.6$ and $b = 1.4$, and so

$$a^2 = c^2 - b^2 = (7.6)^2 - (1.4)^2 = 55.8.$$

Hence

$$a = \sqrt{55.8} = 7.5 \text{ (to one decimal place).}$$

The answer is the top of ladder is 7.5 m (to one decimal place) up the wall.

1 mark

Question 3

$$\begin{aligned} cd^2(3c^3d^4 + 4c^2d) &= cd^2 \times 3c^3d^4 + cd^2 \times 4c^2d \\ &= 3cc^3d^2d^4 + 4cc^2d^2d \\ &= 3c^4d^6 + 4c^3d^3. \end{aligned}$$

The answer is $3c^4d^6 + 4c^3d^3$. 1 mark

Question 4

Multiply each term inside the first bracket by each term inside the second bracket, and add the resulting terms. This gives:

$$\begin{aligned} (3x+4)(2x-5) &= 3x \times 2x + 3x \times (-5) + 4 \times 2x + 4 \times (-5) \\ &= 6x^2 - 15x + 8x - 20 \\ &= 6x^2 - 7x - 20. \end{aligned}$$

The answer is $6x^2 - 7x - 20$. 1 mark

Question 5

Subtract 3 from both sides of the equation to give

$$\frac{2x-3}{x} = 6.$$

Now multiply both sides of this equation by x to give

$$2x - 3 = 6x.$$

Subtract $2x$ from both sides of this equation to give

$$-3 = 4x, \text{ and so } x = -\frac{3}{4}.$$

The answer is $x = -\frac{3}{4}$. 1 mark

Question 6

To factorise $x^2 - 2x - 8$ you must fill in the gaps in the brackets on the right-hand side of the equation

$$x^2 - 2x - 8 = (x + \_\_)(x + \_\_)$$

with two numbers whose product is -8 and whose sum is -2. The pairs of integers whose product is -8 are

$$-1, 8; \quad -8, 1; \quad -2, 4; \quad -4, 2.$$

Of these, only the fourth pair has the sum -2. So the factorisation is

$$x^2 - 2x - 8 = (x - 4)(x + 2).$$

The answer is $(x - 4)(x + 2)$. 1 mark

Question 7

The quadratic equation $ax^2 + bx + c = 0$ has solutions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The quadratic equation $3x^2 - 4x - 2 = 0$ has $a = 3$, $b = -4$ and $c = -2$, so its solutions are

$$\begin{aligned} x &= \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 3 \times (-2)}}{2 \times 3} \\ &= \frac{4 \pm \sqrt{16 + 24}}{6} \\ &= \frac{4 \pm \sqrt{40}}{6} \\ &= \frac{4 \pm 2\sqrt{10}}{6} = \frac{2 \pm \sqrt{10}}{3}. \end{aligned}$$

That is, the solutions are

$$x = \frac{2 - \sqrt{10}}{3} = -0.3874\ldots \quad \text{and} \quad x = \frac{2 + \sqrt{10}}{3} = 1.7207\ldots.$$

The answers are $x = -0.387$ (to three decimal places) $\frac{1}{2}$ mark

and $x = 1.721$ (to three decimal places). $\frac{1}{2}$ mark

Question 8

The line with gradient m and y-intercept c has equation

$$y = mx + c.$$

The line that passes through the points (a, b) and (c, d) has gradient

$$m = \frac{d-b}{c-a}.$$

Therefore, the line through the points $(5, -1)$ and $(-1, 3)$ has gradient

$$m = \frac{3-(-1)}{-1-5} = \frac{4}{-6} = -\frac{2}{3}.$$

Hence the equation of the line is

$$y = -\frac{2}{3}x + c$$

for some constant c. This equation can be rearranged as

$$c = y + \frac{2}{3}x.$$

The line must pass through the point $(5, -1)$, so substituting $x = 5$ and $y = -1$ into this equation gives

$$c = -1 + \frac{2}{3} \times 5 = -1 + \frac{10}{3} = \frac{7}{3}.$$

Therefore the line has equation

$$y = -\frac{2}{3}x + \frac{7}{3}.$$

The answer is $y = -\frac{2}{3}x + \frac{7}{3}$. 1 mark

Question 9

First rewrite each fraction so that the two fractions have the same denominator. You can do this by multiplying the top and bottom of the first fraction by 4, and the top and bottom of the second fraction by 3:

$$\frac{4(x-2)}{12} + \frac{3(2x+5)}{12}.$$

Now add the numerators to give

$$\frac{4(x-2)+3(2x+5)}{12}.$$

Finally, expand the brackets in the numerator to give the answer

$$\frac{4(x-2)+3(2x+5)}{12} = \frac{4x-8+6x+15}{12} = \frac{10x+7}{12}.$$

The answer is $\frac{10x+7}{12}$. 1 mark

Question 10

First apply the index law

$$a^m \times a^n = a^{m+n}$$

to the numerator of the fraction:

$$p^{\frac{7}{2}} p^{\frac{3}{4}} = p^{\frac{7}{2}+\frac{3}{4}} = p^{\frac{17}{4}}.$$

Then apply the index law

$$\frac{a^m}{a^n} = a^{m-n}$$

to give the result

$$\frac{p^{\frac{7}{2}} p^{\frac{3}{4}}}{p^{\frac{5}{4}}} = \frac{p^{\frac{17}{4}}}{p^{\frac{5}{4}}} = p^{\frac{17}{4}-\frac{5}{4}} = p^3.$$

The answer is p^3. 1 mark

Question 11

Using the fraction law

$$\left(\frac{p}{q}\right) / \left(\frac{r}{s}\right) = \frac{p}{q} \times \frac{s}{r}$$

gives

$$\begin{aligned}\left(\frac{14x^5}{15y^{-2}}\right) / \left(\frac{2x^{-2}}{5y^3}\right) &= \frac{14x^5}{15y^{-2}} \times \frac{5y^3}{2x^{-2}} \\ &= \frac{14x^5 \times 5y^3}{15y^{-2} \times 2x^{-2}}\end{aligned}$$

Dividing top and bottom by 2×5 to simplify the numbers gives

$$\frac{7x^5y^3}{3y^{-2}x^{-2}},$$

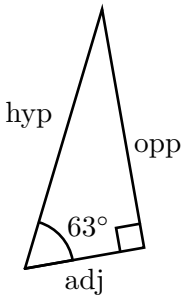
and multiplying top and bottom by x^2y^2 gives

$$\frac{7x^7y^5}{3} = \frac{7}{3}x^7y^5.$$

The answer is $\frac{7}{3}x^7y^5$. 1 mark

Question 12

Label the side lengths of the right-angled triangle as shown on the right, where "opp" stands for "opposite the angle 63°", "adj" stands for "adjacent to the angle 63°" and "hyp" stands for "hypotenuse". (The hypotenuse is the longest side of a right-angled triangle.)



Then

$$\sin 63^\circ = \frac{\text{opp}}{\text{hyp}}, \quad \cos 63^\circ = \frac{\text{adj}}{\text{hyp}}, \quad \text{and} \quad \tan 63^\circ = \frac{\text{opp}}{\text{adj}}$$

The shortest side of this triangle is adj, and hyp is given as 15.3 cm, so you should use the formula for cosine. Rearranging the formula you obtain

$$\text{adj} = \text{hyp} \times \cos 63^\circ.$$

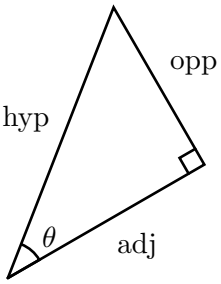
Substituting 15.3 cm for hyp in this equation, and using a calculator, gives

$$\text{adj} = 15.3 \times \cos 63^\circ = 6.946\ldots \text{ cm}.$$

The answer is 6.9 cm to two significant figures. 1 mark

Question 13

Label the side lengths of the right-angled triangle as shown on the right, where "opp" stands for "opposite the angle θ", "adj" stands for "adjacent to the angle θ" and "hyp" stands for "hypotenuse". (The hypotenuse is the longest side of a right-angled triangle.)



Recall the trigonometric rations **sine**, **cosine** and **tangent** given by

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \quad \cos\theta = \frac{\text{adj}}{\text{hyp}}, \quad \text{and} \quad \tan\theta = \frac{\text{opp}}{\text{adj}}.$$

You are given that opp=3.1 cm and adj=6.3 cm, so it follows that

$$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{3.1}{6.3} = 0.492063\ldots.$$

This implies that

$$\theta = \tan^{-1}(0.492063\ldots),$$

and using your calculator you should find that

$$\theta = 26.2\ldots^\circ.$$

So the answer is $\theta = 26^\circ$ to the nearest degree. 1 mark

Question 14

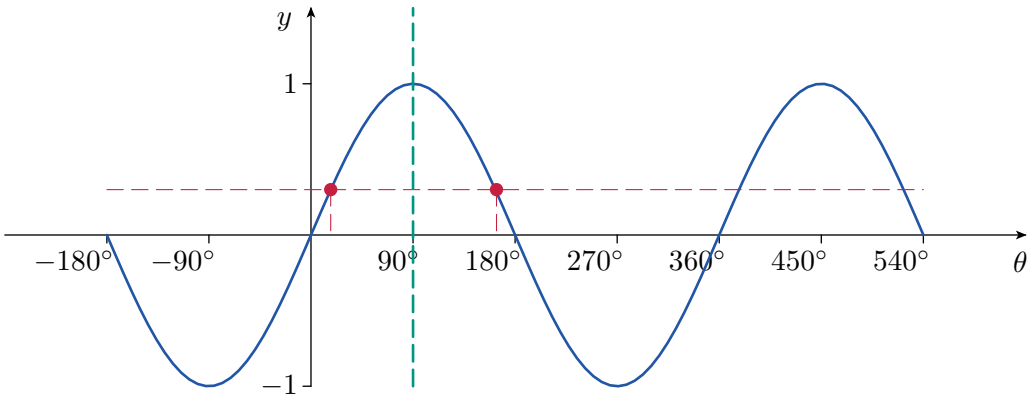
The angle θ is such that $\sin\theta = 0.3$.

The $\sin^{-1}$ button on a calculator gives the value of θ between $0°$ and $90°$:

$$\theta = \sin^{-1} 0.3 = 17.457\ldots{}° .$$

So the first solution is $17°$ to the nearest degree. $\boxed{\frac{1}{2}\text{ mark}}$

To find the value of θ between $90°$ and $180°$ you can use the graph of sine.



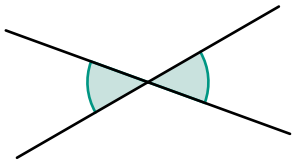
The vertical green dashed line through $90°$ is a line of reflectional symmetry of the graph. It shows that the solution of $\sin\theta = 0.3$ between $90°$ and $180°$ is

$$180° - 17.457\ldots{}° = 162.542\ldots{}° .$$

So the second solution is $\theta = 163°$ to the nearest degree. $\boxed{\frac{1}{2}\text{ mark}}$

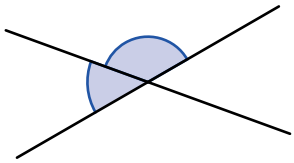
Question 15

When a pair of straight lines cross, the two *opposite* angles (shaded in the diagram on the right) are equal. Since ϕ is opposite the angle $\frac{3}{7}\pi$ it follows that



$$\phi = \frac{3}{7}\pi.$$

When a pair of straight lines cross, the sum of the two adjacent angles (shaded in the diagram on the right) is π radians. (Recall that there are 2π radians in a full turn, and therefore π radians in a half turn. In other words, π radians is equal to $180°$.) Since the angle θ is adjacent to $\phi = \frac{3}{7}\pi$ it follows that



$$\theta + \frac{3}{7}\pi = \pi.$$

Hence

$$\theta = \pi - \frac{3}{7}\pi = \frac{4}{7}\pi.$$

So the answers are $\phi = \frac{3}{7}\pi$ $\boxed{\frac{1}{2}\text{ mark}}$

and $\theta = \frac{4}{7}\pi$. $\boxed{\frac{1}{2}\text{ mark}}$

Question 16

Recall the index laws

$$(a^m)^n = a^{mn} \qquad (a \times b)^n = a^n \times b^n,$$

which are true for any numbers m and n. Let's begin by raising both sides of the equation in the question to the power 4. The left-hand side becomes A^4 and, by the first index law, the right-hand side becomes

$$\left(\left(\frac{B^2}{C^3}\right)^{\frac{1}{4}}\right)^4 = \left(\frac{B^2}{C^3}\right)^{\frac{1}{4}\times 4} = \frac{B^2}{C^3}$$

The equation now says

$$A^4 = \frac{B^2}{C^3}.$$

Multiplying both sides by C^3 gives

$$B^2 = A^4 C^3.$$

Now, let's raise each side of the equation to the power $\frac{1}{2}$. The left-hand side becomes

$$\left(B^2\right)^{\frac{1}{2}} = B^{\frac{1}{2}\times 2} = B.$$

The right-hand side becomes

$$\left(A^4 C^3\right)^{\frac{1}{2}}.$$

By the second index law, this is equal to

$$\left(A^4\right)^{\frac{1}{2}} \left(C^3\right)^{\frac{1}{2}},$$

and by the first index law, this is equal to

$$A^2 C^{\frac{3}{2}}.$$

So the answer is $B = A^2 C^{\frac{3}{2}}$. 1 mark

Question 17

The following operations can be applied to an inequality without changing its meaning.

(1) Add or subtract a number to both sides of the inequality.
(2) Multiply or divide both sides of the inequality by a positive number.
(3) Multiply or divide both sides of the inequality by a negative number, *if you also reverse the inequality sign.*

Let's apply rule (1) by adding 5 to both sides of the inequality. You obtain

$$4x \leq 6x + 8.$$

Now apply rule (1) again, by subtracting $6x$ from both sides of this inequality. You obtain

$$-2x \leq 8.$$

Finally, apply rule (3) by dividing both sides of this inequality by -2, giving

$$x \geq -4.$$

The answer is $x \geq -4$. 1 mark

Question 18

Multiply both sides of the first equation by 2, and both sides of the second equation by 3 to give

$$10x - 6y = -14$$
$$9x - 6y = -27.$$

Now subtract the second of these new equations from the first:

$$(10x - 9x) - (6y - 6y) = -14 - (-27),$$

that is, $x = 13$.

Substitute $x = 13$ into the first of the original equations to give

$$10 \times 13 - 6y = -14,$$

that is, $130 - 6y = -14$.

Solving this gives $y = 24$. So the answer is $x = 13, y = 24$. 1 mark

Question 19

Recall the logarithm laws: $\ln x - \ln y = \ln\left(\frac{x}{y}\right)$ $n \ln x = \ln\left(x^n\right)$.

Using the second law, gives $2\ln 5 = \ln\left(5^2\right) = \ln 25$.

Therefore, $\ln 4 - 2\ln 5 = \ln 4 - \ln 25$.

Using the first law then gives $\ln 4 - 2\ln 5 = \ln\left(\frac{4}{25}\right)$.

The answer is $\ln\left(\frac{4}{25}\right)$. 1 mark

Question 20

Apply ln to both sides of the equation to give $\ln\left(3^t\right) = \ln 15$.

It follows from the logarithm law $n \ln x = \ln\left(x^n\right)$ that $\ln\left(3^t\right) = t \ln 3$.

Hence $t \ln 3 = \ln 15$ and so $t = \frac{\ln 15}{\ln 3}$.

Now, $\ln 15 = 2.708\ldots$ and $\ln 3 = 1.098\ldots$, and hence $t = \dfrac{2.708\ldots}{1.098\ldots} = 2.4649\ldots$.

So the answer is $t = 2.46$ to three significant figures. 1 mark