MST125

Are you ready for MST125?

Instructions

Welcome to the *Are you ready for MST125?* quiz.

This quiz will help you assess whether you have the mathematical skills needed to study *Essential mathematics 2* (MST125) – or, if you are thinking of taking *Essential mathematics 1* (MST124) and MST125 at the same time, whether it is advisable for you to do so.

If you have not studied MST124 and intend to start studying it before or at the same time as MST125, then you should try the *Are you ready for MST124?* quiz before you try this MST125 quiz.

The *Are you ready for MST125?* quiz can also be used to identify topics you need to brush up on or revise in preparation for this module. Most of the questions in this MST125 quiz test your familiarity with topics that are taught in MST124.

There are twenty questions, which you can attempt in any order, and ideally you should try all of them. Keep a note of how long it takes you to complete the quiz. You may need up to an hour.

Solutions are provided on pages 6–18. You should not look at the solutions until you have answered all the questions to the best of your ability. Follow the instructions on page 6 to determine your score, and then consult the Study Advice that is appropriate to your score and to how MST125 features in your study programme.

Attempt this quiz using pen, paper and a basic scientific calculator (one that cannot carry out algebraic manipulation, or algebraic or numerical differentiation and integration).

Good luck!

Question 1

What is the value of the expression

$$\frac{x^2 - 3x + 1}{(2 - 3x)(x - 1)^2}$$

when $x = -\frac{1}{2}$? Give your answer as a fraction.
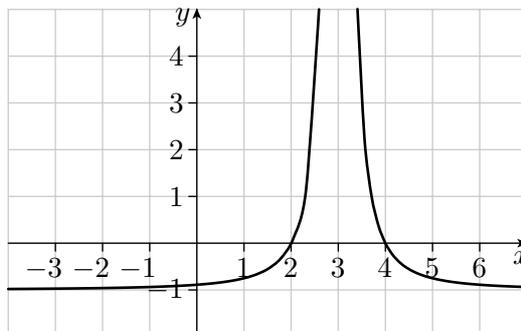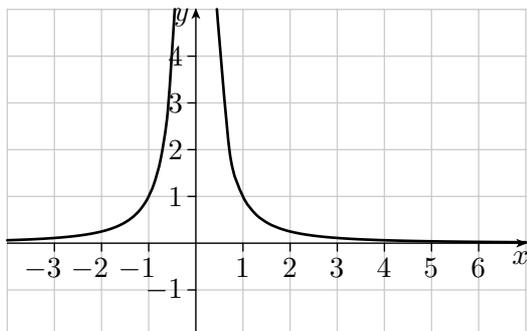
Question 2

Express the sum of fractions

$$\frac{3}{x - 1} + \frac{2x}{x^2 - 1} - \frac{1}{x}$$

as a single fraction.

Question 3

The graphs of $y = \dfrac{1}{x^2}$ and $y = \dfrac{1}{(x - a)^2} + b$, where a and b are integers, possibly negative, are shown below.

What are the values of a and b?

Question 4

If $f(x) = \frac{1}{2}x^2$ and $g(x) = 4x - 7$, what is the rule of the composite function $g \circ f$?

Question 5

What is the rule of the inverse function of $f(x) = \frac{1}{3}x + 4$?

Question 6

Find all solutions between 0 and 2π of the equation

$$\sin \theta = -\frac{\sqrt{3}}{2}.$$

Give exact answers in radians.

Question 7

Find the magnitude of the vector $-3\,\mathbf{i} - 5\,\mathbf{j}$, and the angle between $0°$ and $360°$ measured anticlockwise from the positive x-direction to the direction of this vector. Here $\mathbf{i}$ and $\mathbf{j}$ are the Cartesian unit vectors in the positive directions of the x- and y-axes, respectively.

Give the magnitude to one decimal place and the angle to the nearest degree.

Question 8

The vector $\mathbf{v}$ shown below has magnitude 4. Find its component form, giving the components to one decimal place.

The vectors $\mathbf{i}$ and $\mathbf{j}$ are the Cartesian unit vectors in the positive directions of the x- and y-axes, respectively.
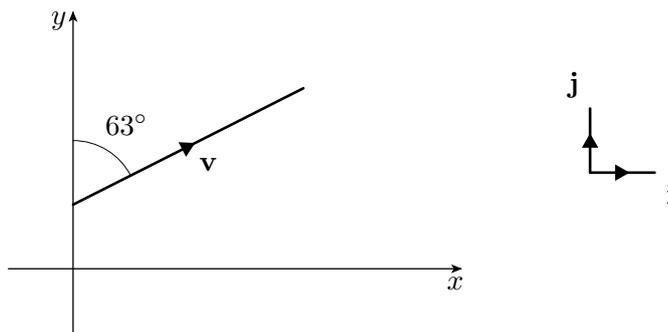
Question 9

Calculate the missing element in the product matrix

$$\begin{pmatrix} 0 & 1 & -3 & \frac{4}{5} \\ -1 & 0 & 2 & \frac{1}{2} \\ 21 & -7 & \frac{1}{3} & 0 \end{pmatrix} \begin{pmatrix} 1 & -\frac{1}{3} \\ -2 & 2 \\ -1 & 0 \\ 5 & 1 \end{pmatrix} = \begin{pmatrix} 5 & \frac{14}{5} \\ -\frac{1}{2} & \boxed{} \\ \frac{89}{3} & -22 \end{pmatrix}.$$

Question 10

If $\mathbf{A} = \begin{pmatrix} -6 & -1 \\ 2 & 1 \end{pmatrix}$, what is $\mathbf{A}^{-1}$?

Question 11

Differentiate the function

$$f(x) = 5x^{2/3} + x^{-2}.$$

Question 12

What is the gradient of the graph of the function

$$g(x) = 7x^2 \ln x$$

at the point where $x = 3$? Give your answer to one decimal place.

Question 13

Find the stationary points of the function

$$f(x) = 7x^3 - \tfrac{1}{2}x^2,$$

and determine the nature of each stationary point.

Question 14

Find the derivative

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{2x-3}{(x+1)(x+2)}\right).$$

Question 15

Find the derivative

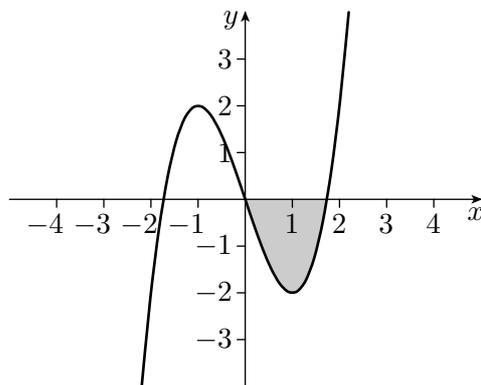$$\frac{\mathrm{d}}{\mathrm{d}x}\left(e^{\cos 2x}\right).$$

Question 16

Find the integral

$$\int \frac{1}{2x-3}\,\mathrm{d}x.$$

Question 17

The graph of the function $f(x) = x^3 - 3x$ is shown below. What is the area of the shaded region? Give your answer as a fraction.

Question 18

Find the integral

$$\int e^{\sin 2x} \cos 2x\,\mathrm{d}x.$$

Question 19

Find the value of the definite integral

$$\int_{\pi/4}^{\pi/2} x \cos x \, \mathrm{d}x$$

to three significant figures.

Question 20

By using the substitution $u = x - 1$, find the integral

$$\int \frac{3x - 7}{(x - 1)^2} \, \mathrm{d}x \, .$$

Solutions

Well done for completing the *Are you ready for MST125?* quiz.

Please ensure you have completed all the questions to the best of your ability before opening this document. Mark your answers according to the solutions given below. There is one mark for each question.

These solutions contain references to relevant parts of MST124 and they should help if you have studied, or are currently studying, MST124 and need a hint or a reminder. The references should also help you work out which parts of MST124 you might need to revise, and they are summarised in a table on pages 25 and 26 for convenience.

After you have calculated your overall score, go to the Study advice, which starts on page 19.

Question 1

To calculate the value of the expression, substitute in the negative number $-\frac{1}{2}$. Remember to enclose $-\frac{1}{2}$ in brackets. The resulting expression contains fractions and negative numbers, so carry out the required arithmetic carefully and systematically to avoid errors.

When $x = -\frac{1}{2}$,

$$
\frac{x^2 - 3x + 1}{(2 - 3x)(x - 1)^2} = \frac{\left(-\frac{1}{2}\right)^2 - 3\left(-\frac{1}{2}\right) + 1}{\left(2 - 3\left(-\frac{1}{2}\right)\right)\left(\left(-\frac{1}{2}\right) - 1\right)^2}
$$

$$
= \frac{\frac{1}{4} + \frac{3}{2} + 1}{\left(2 + \frac{3}{2}\right)\left(-\frac{3}{2}\right)^2}
$$

$$
= \frac{\frac{11}{4}}{\frac{7}{2} \times \frac{9}{4}}
$$

$$
= \frac{\frac{11}{4}}{\frac{63}{8}}
$$

$$
= \frac{11}{4} \times \frac{8}{63}
$$

$$
= \frac{22}{63}.
$$

Answer: $\dfrac{22}{63}$.

References: Activity 6 on page 17 of MST124 Book A (Unit 1).
Suggested revision: MST124 Unit 1, Subsection 1.2 Working with numbers.

Question 2

Factorise $x^2 - 1$ and use the factorisation to write the fractions with the same denominator, by multiplying the top and bottom of each fraction by an appropriate expression.

This gives

$$\frac{3}{x-1} + \frac{2x}{x^2-1} - \frac{1}{x}$$

$$= \frac{3}{x-1} + \frac{2x}{(x-1)(x+1)} - \frac{1}{x}$$

$$= \frac{3x(x+1)}{x(x-1)(x+1)} + \frac{2x \times x}{x(x-1)(x+1)} - \frac{x^2-1}{x(x-1)(x+1)}$$

$$= \frac{3x(x+1) + 2x^2 - (x^2-1)}{x(x-1)(x+1)}$$

$$= \frac{3x^2 + 3x + 2x^2 - x^2 + 1}{x(x^2-1)}$$

$$= \frac{4x^2 + 3x + 1}{x(x^2-1)}.$$

Answer: $\dfrac{4x^2 + 3x + 1}{x(x^2-1)}$.

References: Example 15 on page 51 of MST124 Book A (Unit 1).
Suggested revision: MST124 Unit 1, Subsection 3.3 Algebraic fractions.

Question 3

Suppose that f is a function and c is a constant. Then to obtain the graph of $y = f(x) + c$ you translate the graph of $y = f(x)$ up by c units (the translation is down if c is negative). To obtain the graph of $y = f(x - c)$ you translate the graph of $y = f(x)$ to the right by c units (the translation is to the left if c is negative).

The graph on the right is the graph of $y = \dfrac{1}{x^2}$ translated down by 1 unit and to the right by 3 units.

Translating the graph of $y = \dfrac{1}{x^2}$ down by 1 unit gives the graph of $y = \dfrac{1}{x^2} + (-1).$

Then translating *this* graph to the right by 3 units gives the graph of $y = \dfrac{1}{(x-3)^2} + (-1).$

So the graph on the right is the graph of $y = \dfrac{1}{(x-3)^2} + (-1)$, that is, $y = \dfrac{1}{(x-3)^2} - 1.$ Hence $a = 3$ and $b = -1$.

Answer: $a = 3$, $b = -1$.

Question 4

By definition, $(g \circ f)(x) = g(f(x))$. so

$$(g \circ f)(x) = g(f(x))$$
$$= 4\left(\tfrac{1}{2}x^2\right) - 7$$
$$= 2x^2 - 7.$$

Answer: $(g \circ f)(x) = 2x^2 - 7$.

Question 5

To find the rule of the inverse of f, rearrange the equation $\frac{1}{3}x + 4 = y$ to express x in terms of y. This gives

$$\tfrac{1}{3}x + 4 = y$$
$$\tfrac{1}{3}x = y - 4$$
$$x = 3y - 12.$$

Hence the rule of f^{-1} is

$$f^{-1}(y) = 3y - 12,$$

that is,

$$f^{-1}(x) = 3x - 12.$$

Answer: $f^{-1}(x) = 3x - 12$.

Question 6

You can use your calculator to find the value of $\sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$ (making sure that your calculator is set to radians), but this value is not between 0 and 2π. To find the solutions between 0 and 2π, use the value of $\sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)$ together with the symmetry of the graph of the sine function.

A sketch of the graph of the sine function shows that there are two solutions between 0 and 2π.
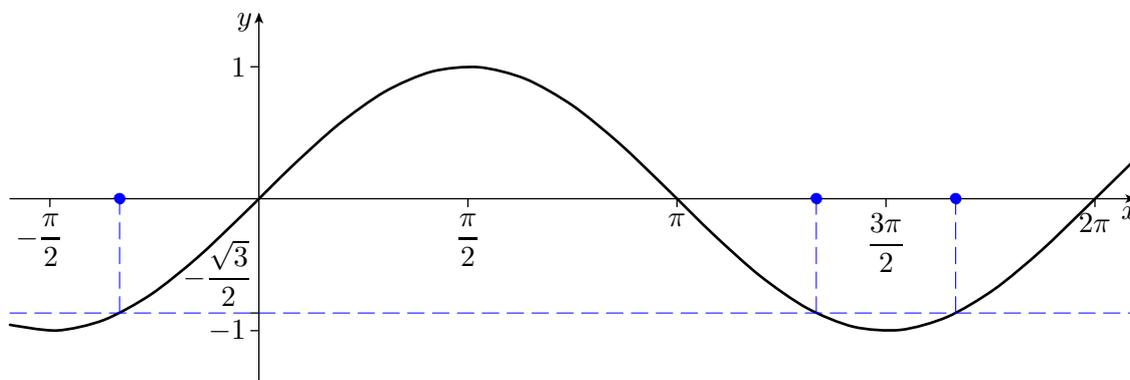
A calculator gives $\sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right) = -\dfrac{\pi}{3}$, which is the negative solution shown in the diagram above. By the symmetry of the graph, it follows that the two solutions between 0 and 2π are

$$\pi + \frac{\pi}{3} = \frac{4\pi}{3} \quad \text{and} \quad 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}.$$

Answer: The solutions are $\dfrac{4\pi}{3}, \dfrac{5\pi}{3}$.

References: Example 8 on page 51 of MST124 Book B (Unit 4).
Trigonometric equations, MST124 *Handbook*, page 36.
Suggested revision: MST124 Unit 4, Subsection 2.5 Solving simple trigonometric equations.
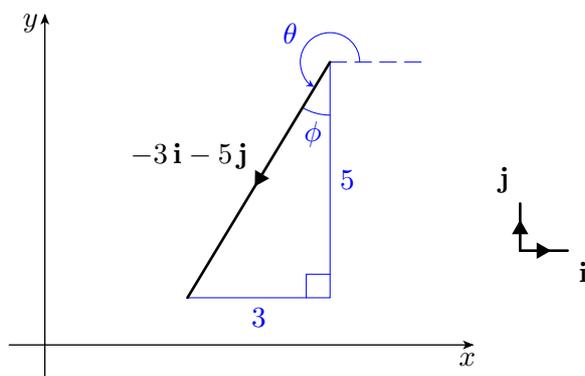
Question 7

The magnitude of the vector $a\,\mathbf{i} + b\,\mathbf{j}$ is given by $\sqrt{a^2 + b^2}$.

Therefore the magnitude of the vector $-3\,\mathbf{i} - 5\,\mathbf{j}$ is

$$\sqrt{(-3)^2 + (-5)^2} = \sqrt{9 + 25} = \sqrt{34} = 5.830\ldots = 5.8 \text{ (to 1 d.p.)}.$$

To calculate the required angle, start by sketching a right-angled triangle whose hypotenuse is the given vector and whose shortest sides are parallel to the x- and y-axes.

The required angle is the angle θ in the diagram below.

To calculate the angle θ, we start by calculating the angle ϕ. From the diagram,

$$\tan \phi = \tfrac{3}{5},$$

so

$$\phi = \tan^{-1} \tfrac{3}{5} = 30.963\ldots^\circ = 31^\circ \text{ (to the nearest degree)}.$$

Hence the required angle is

$$\theta = 270^\circ - \phi = 239^\circ \text{ (to the nearest degree)}.$$

Answer: The magnitude is 5.8 (to 1 d.p.) and the angle is 239° (to the nearest degree).

References: Example 18 on page 174 of MST124 Book B (Unit 5).
Converting vectors from component form to magnitude and direction, and vice versa, MST124 *Handbook*, page 42.
Suggested revision: MST124 Unit 5, Subsection 6.4 Converting from component form to magnitude and direction, and vice-versa.

Question 8

If the vector $\mathbf{v}$ makes the angle θ with the positive x-direction, then
$\mathbf{v} = |\mathbf{v}| \cos\theta\, \mathbf{i} + |\mathbf{v}| \sin\theta\, \mathbf{j}$.

The angle from the positive x-direction to the direction of the given vector is
$90° - 63° = 27°$. So the component form of the vector is

$$4\cos 27°\, \mathbf{i} + 4\sin 27°\, \mathbf{j} = 3.564\ldots\, \mathbf{i} + 1.815\ldots\, \mathbf{j}$$
$$= 3.6\, \mathbf{i} + 1.8\, \mathbf{j} \text{ (to 1 d.p.).}$$

Answer: $\mathbf{v} = 3.6\, \mathbf{i} + 1.8\, \mathbf{j}$ (to 1 d.p.).

References: Example 19 on page 176, and Activity 48 on page 179 of
MST124 Book B (Unit 5).
Converting vectors from component form to magnitude and direction,
and vice versa, MST124 *Handbook*, page 42.
Suggested revision: MST124 Unit 5, Subsection 6.4 Converting from
component form to magnitude and direction, and vice-versa.

Question 9

When multiplying two matrices $\mathbf{A}$ and $\mathbf{B}$, the element in the ith row, jth
column of $\mathbf{AB}$ is obtained by multiplying corresponding entries in the ith
row of $\mathbf{A}$ and the jth column of $\mathbf{B}$ and adding the results.

The missing element is in the second row, second column of the product
matrix, so it is obtained by multiplying corresponding entries in the second
row of $\mathbf{A}$ and the second column of $\mathbf{B}$ and adding the results. This gives

$$-1 \times \left(-\frac{1}{3}\right) + 0 \times 2 + 2 \times 0 + \frac{1}{2} \times 1$$
$$= \frac{1}{3} + \frac{1}{2}$$
$$= \frac{5}{6}.$$

Answer: $\dfrac{5}{6}$.

References: Example 1 on pages 231–232 of MST124 Book C (Unit 9).
Matrix multiplication, MST124 *Handbook*, page 55.
Suggested revision: MST124 Unit 9, Subsection 1.4 Matrix multiplication.

Question 10

The inverse of a 2×2 matrix $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is

$$\frac{1}{\det \mathbf{A}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

Here $\det \mathbf{A} = -6 \times 1 - (-1) \times 2 = -6 + 2 = -4$, so the inverse of $\mathbf{A}$ is

$$\mathbf{A}^{-1} = -\frac{1}{4} \begin{pmatrix} 1 & 1 \\ -2 & -6 \end{pmatrix} = \begin{pmatrix} -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{2} & \frac{3}{2} \end{pmatrix}.$$

Answer: $\begin{pmatrix} -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{2} & \frac{3}{2} \end{pmatrix}.$

References: Example 4 on pages 255–256 of MST124 Book C (Unit 9).
Matrix inverses and determinants, MST124 *Handbook*, page 56.
Suggested revision: MST124 Unit 9, Section 3 The inverse of a matrix.

Question 11

Each term contains a power function; to differentiate a power function multiply by the power, then reduce the power by one. This gives

$$f'(x) = \frac{2}{3} \times 5x^{-\frac{2}{3}-1} + (-2) \times x^{-2-1}$$

$$= \frac{10}{3} x^{-1/3} - 2x^{-3}.$$

Answer: $f'(x) = \frac{10}{3} x^{-1/3} - 2x^{-3}.$

References: Example 4 on page 247 of MST124 Book B (Unit 6).
Power functions, MST124 *Handbook*, page 44.
Constant multiple rule and sum rule for derivatives,
MST124 *Handbook*, page 44.
Suggested revision: MST124 Unit 6, Section 2 Finding derivatives of simple functions.

Question 12

The derivative of a function f is the function f' such that

$$f'(x) = \text{ gradient of the graph of } f \text{ at the point } (x, f(x)).$$

To find the derivative of the function g in this question, use the product rule. This gives

$$g'(x) = 7x^2 \times \frac{1}{x} + \ln x \times 14x$$
$$= 7x + 14x \ln x$$
$$= 7x(1 + 2\ln x).$$

So the gradient of the graph of g at the point where $x = 3$ is

$$7 \times 3(1 + 2\ln 3) = 21(1 + 2\ln 3) = 67.141\ldots = 67.1 \text{ (to 1 d.p.).}$$

Answer: 67.1 (to 1 d.p.).

References: Activity 8 on page 238 of MST124 Book B (Unit 6).
    Derivatives, MST124 *Handbook*, page 44.
    Differentiation rules, MST124 *Handbook*, pages 44 and 48.
Suggested revision: MST124 Unit 6, Subsection 1.4 Derivatives,
MST124 Unit 7, MST124 Subsection 2.1 Product rule.

Question 13

A stationary point of a function f is a value of x at which $f'(x) = 0$.

To determine the nature of a stationary point x_0, look at the sign of $f'(x)$ on intervals immediately to the left of x_0 and immediately to the right of x_0, to determine whether f is increasing or decreasing on those intervals. (Alternatively, use the second derivative test.)

The derivative of the given function f is

$$f'(x) = 3 \times 7x^{3-2} - 2 \times \tfrac{1}{2}x^{2-1}$$
$$= 21x^2 - x$$
$$= x(21x - 1).$$

Solving the equation $f'(x) = 0$ gives

$$x(21x - 1) = 0,$$

that is

$$x = 0 \quad \text{or} \quad x = \tfrac{1}{21}.$$

Hence the stationary points are 0 and $\frac{1}{21}$.

A table of signs for $f'(x)$ is below.

	$(-\infty, 0)$	0	$(0, \frac{1}{21})$	$\frac{1}{21}$	$(\frac{1}{21}, \infty)$
x	$-$	0	$+$	$+$	$+$
$21x - 1$	$-$	$-$	$-$	0	$+$
$f'(x)$	$+$	0	$-$	0	$+$

The function f is increasing on the left of 0 and decreasing on the right, so 0 is a maximum.

The function f is decreasing on the left of $\frac{1}{21}$ and increasing on the right, so $\frac{1}{21}$ is a minimum.

Answer: The stationary point at $x = 0$ is a maximum.

The stationary point at $x = \frac{1}{21}$ is a minimum.

References: Example 8 on page 266 of MST124 Book B (Unit 6).
Stationary points, MST124 *Handbook*, page 45.
First derivative test (for determining the nature of a stationary point), MST124 *Handbook*, page 45.
Suggested revision: MST125 Unit 6, Subsection 4.2 Stationary points.

Question 14

Here we must differentiate an expression of the form $\dfrac{f(x)}{g(x)}$, where
$f(x) = 2x - 3$ and $g(x) = (x+1)(x+2)$. We need to use the quotient rule.
To make it easier to differentiate the denominator, we multiply out the brackets.

We have $\dfrac{d}{dx}(f(x)) = 2$ and $\dfrac{d}{dx}(g(x)) = \dfrac{d}{dx}(x^2 + 3x + 2) = 2x + 3$.

By the quotient rule,

$$\frac{d}{dx}\left(\frac{2x-3}{(x+1)(x+2)}\right) = \frac{d}{dx}\left(\frac{2x-3}{x^2+3x+2}\right)$$

$$= \frac{(x^2+3x+2)\times 2 - (2x-3)(2x+3)}{(x^2+3x+2)^2}$$

$$= \frac{2x^2+6x+4-4x^2+9}{(x+1)^2(x+2)^2}$$

$$= \frac{-2x^2+6x+13}{(x+1)^2(x+2)^2}.$$

Answer: $\dfrac{-2x^2+6x+13}{(x+1)^2(x+2)^2}$

References: Example 3 on pages 19–20 of MST124 Book C (Unit 7).
Quotient rule, MST124 *Handbook*, page 48.
Suggested revision: MST124 Unit 7, Subsection 2.2 Quotient rule.

Question 15

Use the chain rule, together with the rule for differentiating a function of a linear expression, which is

$$\frac{d}{dx}\left(f(ax+b)\right) = af'(ax+b).$$

Applying the chain rule gives

$$\frac{d}{dx}\left(e^{\cos 2x}\right) = e^{\cos 2x} \times \frac{d}{dx}\left(\cos 2x\right)$$

$$= e^{\cos 2x} \times 2(-\sin 2x)$$

$$= -2e^{\cos 2x}\sin 2x\,.$$

Here the derivative of $\cos 2x$ was found by using the fact that it is a function of a linear expression.

Answer: $-2e^{\cos 2x}\sin 2x\,.$

References: Example 12 on page 34 of MST124 Book C (Unit 7).
Chain rule, MST124 *Handbook*, page 48.
Derivative of a function of a linear expression,
MST124 *Handbook*, page 48.
Suggested revision: MST124 Unit 7, Subsection 2.3 Chain rule,
MST124 Unit 7, Subsection 2.4 Differentiating functions of linear expressions.

Question 16

The integrand is a function of a linear expression. You can use the fact that if f is a function with antiderivative F, and a and b are constants with $a \neq 0$, then

$$\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + c,$$

where c is an arbitrary constant. This gives

$$\int \frac{1}{2x-3}\,dx = \tfrac{1}{2}\ln|2x-3| + c,$$

where c is an arbitrary constant.

Alternatively, you can use integration by substitution. Let $u = 2x - 3$; then $\frac{du}{dx} = 2$. So

$$\int \frac{1}{2x-3}\,dx = \tfrac{1}{2}\int \frac{1}{u}\,du$$

$$= \tfrac{1}{2}\ln|u| + c$$

$$= \tfrac{1}{2}\ln|2x-3| + c,$$

where c is an arbitrary constant.

Answer: $\tfrac{1}{2}\ln|2x-3| + c\,.$

References: Example 17 on page 167 of MST124 Book C (Unit 8).

Integration by substitution, MST124 *Handbook*, page 52.

Suggested revision: MST124 Unit 8, Section 3 Integration by substitution.

Question 17

We begin by finding the x-coordinates of the points at which the graph crosses the horizontal axis. These are the values of x that satisfy the equation

$$0 = x^3 - 3x,$$

that is

$$0 = x(x^2 - 3).$$

Solving this equation gives $x = 0$ and $x = \pm\sqrt{3}$.

The *signed* area between the graph and the x-axis from $x = 0$ to $x = \sqrt{3}$ is given by

$$
\int_0^{\sqrt{3}} (x^3 - 3x)\,\mathrm{d}x = \left[\frac{x^4}{4} - \frac{3x^2}{2} \right]_0^{\sqrt{3}}
$$
$$
= \frac{9}{4} - \frac{9}{2}
$$
$$
= -\frac{9}{4}.
$$

Hence the area of the shaded region is $\frac{9}{4}$.

Answer: $\dfrac{9}{4}$.

References: Example 2, page 112 and Example 3, page 117, of MST124 Book C (Unit 8).

Signed areas and definite integrals, MST124 *Handbook*, page 50.

Fundamental theorem of calculus, MST124 *Handbook*, page 51.

Suggested revision: MST124 Unit 8, Section 1 Areas, signed areas and definite integrals, MST124 Unit 7 Subsection 4.2 Antiderivatives of power functions.

Question 18

Use integration by substitution.

Let $u = \sin 2x$; then $\dfrac{\mathrm{d}u}{\mathrm{d}x} = 2\cos 2x$. So

$$\int e^{\sin 2x} \cos 2x \, \mathrm{d}x = \frac{1}{2} \int e^u \, \mathrm{d}u$$

$$= \frac{1}{2} e^u + c$$

$$= \frac{1}{2} e^{\sin 2x} + c,$$

where c is an arbitrary constant.

  Answer: $\frac{1}{2} e^{\sin 2x} + c$.

References: Example 127 on page 1587 of MST124 Book C (Unit 8).
                 Integration by substitution, MST124 *Handbook*, page 52.
Suggested revision: MST124 Unit 8, Section 3 Integration by substitution.

Question 19

Use integration by parts, and remember to set your calculator to radian mode.

Integration by parts gives

$$\int x \cos x \, \mathrm{d}x = x \sin x - \int \sin x \, \mathrm{d}x$$

$$= x \sin x + \cos x + c,$$

where c is an arbitrary constant.

It follows that

$$\int_{\pi/4}^{\pi/2} x \cos x \, \mathrm{d}x = \left[x \sin x + \cos x \right]_{\pi/4}^{\pi/2}$$

$$= \frac{\pi}{2} \sin \frac{\pi}{2} + \cos \frac{\pi}{2} - \left(\frac{\pi}{4} \sin \frac{\pi}{4} + \cos \frac{\pi}{4} \right)$$

$$= \frac{\pi}{2} \times 1 + 0 - \left(\frac{\pi}{4} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right)$$

$$= \frac{\pi}{2} - \frac{1}{\sqrt{2}} \left(\frac{\pi}{4} + 1 \right)$$

$$= 0.308\,32\ldots$$

$$= 0.308 \text{ (to 3 s.f.)}.$$

  Answer: 0.308 (to 3 s.f.).

References: Example 23 on page 178 and Examples 28 (pages 185–186)
                 and 29 (page 186) of MST124 Book C (Unit 8).
                 Integration by parts, MST124 *Handbook*, page 53.
Suggested revision: MST124 Unit 8, Section 4 Integration by parts.

Question 20

Before you can carry out the integration, you have to express all parts of the integral in terms of u. Use the fact that $u = x - 1$ to express the numerator $3x - 7$ in terms of u.

Let $u = x - 1$; then $\dfrac{\mathrm{d}u}{\mathrm{d}x} = 1$. Also, $x = u + 1$, so

$$3x - 7 = 3(u + 1) - 7 = 3u - 4.$$

Hence

$$
\begin{aligned}
\int \frac{3x - 7}{(x - 1)^2}\,\mathrm{d}x &= \int \frac{3u - 4}{u^2}\,\mathrm{d}u \\
&= \int \left(\frac{3}{u} - \frac{4}{u^2} \right)\,\mathrm{d}u \\
&= 3\ln |u| + 4u^{-1} + c \\
&= 3\ln |x - 1| + \frac{4}{x - 1} + c,
\end{aligned}
$$

where c is an arbitrary constant.

Answer: $3\ln |x - 1| + \dfrac{4}{x - 1} + c$.

References: Activity 33 on page 176 of MST124 Unit 8.

Integration by substitution, MST124 *Handbook*, page 52.

Suggested revision: MST124 Unit 8, Subsection 3.5 Finding more complicated integrals by substitution.

Study advice

This section contains advice that depends on your overall score and on how MST125 features in your study plans.

Please choose the option that is appropriate to you:

- If you have already studied MST124, go to page 20.

- If are currently studying MST124, go to page 21.

- If you are considering studying MST125 alongside MST124, go to page 22.

- If you do not intend to study MST124, go to page 23.

If you have already studied MST124

The advice in this section applies if you have finished studying MST124. If that is the case, you would be expected to have a good understanding of most of the topics in the quiz, even if your understanding of some of them is a little rusty, perhaps because you studied MST124 some time ago.

The table on pages 25 and 26 shows which MST124 units are covered in each question, and it can help you determine what topics to focus on if the advice for your score suggests you should do some MST124 revision.

If your score is 15 or more out of 20, you should be ready for MST125. However, the more confident and fluent you are with key topics such as algebra, functions, vectors, matrices and calculus, the easier you'll find it to study MST125. It would do no harm to brush up on any topics you feel less confident with, and to keep practising your skills in the key topics.

If your score is between 10 and 14 out of 20, your score suggests that as long as you can find time to brush up on topics for which you didn't achieve full marks then you are probably ready for MST125. If it's been a long time since you studied MST124, you may need to revise the content of MST124, and practise the techniques there until you become fluent. If you are not confident that you have time to do this before you start MST125, you should consider delaying your start of MST125.

If your score is between 5 and 9 out of 20, then you've achieved good results in some of the questions on this quiz. However, your overall score suggests that you might find MST125 a challenge, as MST125 assumes you are already fluent with algebra, functions, vectors, matrices and calculus.

If your score is 4 or fewer out of 20, then you are probably not ready to start MST125.

If you would like to discuss whether you are ready for MST125, please contact Student Recruitment and Support (tel: +44 (0)300 303 5303, email `general-enquiries@open.ac.uk`), who, if necessary, will refer you to an educational adviser. It would be helpful to tell them your score on the quiz and how long it took you to complete it.

If you are currently studying MST124

If you have studied only the first few units of MST124, it may be wise to repeat this quiz once you have finished studying up to at least MST124 Unit 6: MST125 assumes you already have good skills in algebra, functions, vectors, matrices and calculus.

If you are at least halfway through MST124, your performance on the questions involving topics that you have covered already will be more significant than your overall score.

The table on pages 25 and 26 shows which topics are covered in each question, with references to MST124 units. You might want to work out what percentage of questions you got right out of those on topics that are covered in the MST124 units you have already studied.

Some of the advice on studying MST125 alongside MST124 might also be relevant to you – have a look on page 22.

If you got around 75% or more, then you've achieved an excellent result, and your score suggests that you should be ready for MST125.

If you got around 50% – 75%, then you've achieved a good result, and your score suggests that you are probably ready for MST125. However, you may need to find time to do some more work on topics for which you didn't achieve full marks, and you may like to return to the *Are you ready for MST125?* quiz later to check that your improvement in those topics is on track.

If you got around 25% – 50%, you've achieved good results in some of the questions. However, your score suggests that you might find MST125 a challenge. You are likely to need to find time to do some more work on topics for which you didn't achieve full marks. You may like to return to the *Are you ready for MST125?* quiz later to check that your improvement in those topics is on track.

If you got less than around 25%, you are probably not ready to start MST125.

If you would like to discuss whether you are ready for MST125, please contact Student Recruitment and Support (tel: +44 (0) 300 303 5303, email `general-enquiries@open.ac.uk`), who, if necessary, will refer you to an educational adviser. It would be helpful to tell them your score on the quiz and how long it took you to complete it.

If you are considering studying MST125 alongside MST124

If you are planning to start MST124 at the same time as MST125, then it is likely that there are topics on the quiz you have not met yet, because you will meet them in your study of MST124.

It would be an advantage to recognise some of the topics in this quiz even if you do not fully remember them, or to be able to answer the questions after a quick reminder. However, even if your score on this quiz is low, you should not immediately rule out the possibility of studying MST124 and MST125 at the same time. Instead, you should consider two very important questions:

(a) Are you *very* confident that you are mathematically well prepared to study MST124? For help answering this question, visit `http://mathschoices.open.ac.uk/` and take the *Are you ready for MST124?* quiz.

(b) Do you have 18 hours *every* week to devote to your combined study of MST124 and MST125?

If the answer to both questions is yes, then, provided you are willing to accept a challenge and work hard, you may wish to study the two modules together.

Preparation before the modules start is crucial, and we recommend you consult `http://mathschoices.open.ac.uk/` and follow the advice there to ensure you are fully prepared for MST124.

If you would like to discuss whether you can take MST124 and MST125 at the same time, please contact Student Recruitment and Support (tel: +44 (0)300 303 5303, email `general-enquiries@open.ac.uk`), who, if necessary, will refer you to an educational adviser. It would be helpful to tell them your score on the *Are you ready for MST124?* quiz and how long it took you to complete it.

If you do not intend to study MST124

The study advice in this section applies if you are not planning to study MST124 at all before studying MST125. If that is the case, you would be expected to have a good understanding of most of the topics in the quiz from another source, such as prior study elsewhere, even if you are a little rusty on some of them. When you have registered on MST125, and the MST125 website opens, you will also have access to PDF versions of the MST124 units.

The table on page 24 shows what topics are covered in which quiz question. Use this to determine any topics that you may wish to focus on if the advice for your score suggests you should do some revision. On page 26 you will find some suggestions of resources that you can use for practice and revision.

If your score is 15 or more out of 20, your score suggests that you should be ready for MST125. However, the more confident and fluent you are with key topics such as algebra, functions, vectors, matrices and calculus, the easier you'll find it to study MST125. It would do no harm to brush up on any topics you feel less confident with, and to keep practising your skills in the key topics.

If your score is between 10 and 14 out of 20, your score suggests that as long as you can find time to brush up on topics for which you didn't achieve full marks then you are probably ready for MST125. The content of MST125 assumes you already have good skills in algebra, functions, vectors, matrices and calculus, so take time to practise these as much as possible.

If your score is between 5 and 9 out of 20, you've achieved good results in some of the questions on this quiz. However, your overall score suggests that you might find MST125 a challenge, as the module assumes you already have fluent skills in algebra, functions, vectors, matrices and calculus. You should consider studying MST124 first.

If your score is 4 or fewer out of 20, you are probably not ready to start MST125. You should consider studying MST124 first.

If you would like to discuss whether you are ready for MST125, please contact Student Recruitment and Support (tel: +44 (0) 300 303 5303, email `general-enquiries@open.ac.uk`), who, if necessary, will refer you to an educational adviser.

If you are not planning to study MST124 at all, but you wish to study MST125, then it is crucial that you undertake independent study to brush up and practise your mathematical skills. We provide a list of suggested resources on page 26, but if you need to make heavy use of these resources, we recommend that you instead consider studying MST124. Whatever resources you choose, after studying them you should repeat the MST125 quiz.

Preparation for MST125

Topics covered in the quiz

Your quiz performance can be used to give you more information than is given by just the total score. Looking back at how you did on individual questions can help you to identify topics on which you may need to brush up. The questions cover six main areas, as in the table below.

Quiz questions	Topic
1, 2	Algebra
3, 4, 5	Functions (graphs, composition, inverses)
6	Trigonometry
7, 8, 9, 10	Vectors and matrices
11, 12, 13, 14, 15	Differentiation (gradients, product rule, quotient rule, composite rule, stationary points)
16, 17, 18, 19, 20	Integration (by substitution, by parts, definite and indefinite integrals, areas)

MST124 units covered in the quiz

This table can help you identify which parts of MST124 you should focus on if your quiz score suggests you should do some revision.

If you'd like to return to the quiz, go to page 2. If you'd like to return to the solutions, go to page 6. If you'd like to return to the Study advice, go to page 19.

Question	MST124 reference
1	Activity 6 on page 17 of MST124 Book A (Unit 1). Suggested revision: MST124 Unit 1, Subsection 1.2 Working with numbers.
2	Example 15 on page 51 of MST124 Book A (Unit 1). Suggested revision: MST124 Unit 1, Subsection 3.3 Algebraic fractions.
3	Activity 20 on page 237 of MST124 Book A (Unit 3). Translating graphs vertically and horizontally, MST124 *Handbook*, page 28. Suggested revision: MST124 Unit 3, Subsection 2.1 Translating the graphs of functions.
4	Example 4 on page 252 of MST124 Book A (Unit 3). Composite functions, MST124 *Handbook*, page 26. Suggested revision: MST124 Unit 3, Subsection 3.2 Composite functions.
5	Example 6 on pages 258–259 of MST124 Book A (Unit 3). Inverse functions, MST124 *Handbook*, page 26. Suggested revision: MST124 Unit 3, Subsection 3.3 Inverse functions.
6	Example 8 on page 51 of MST124 Book B (Unit 4). Trigonometric equations, MST124 *Handbook*, page 36. Suggested revision: MST124 Unit 4, Subsection 2.5 Solving simple trigonometric equations.
7	Example 18 on page 174 of MST124 Book B (Unit 5). Converting vectors from component form to magnitude and direction, and vice-versa, MST124 *Handbook*, page 42. Suggested revision: MST124 Unit 5, Subsection 6.4 Converting from component form to magnitude and direction, and vice-versa.
8	Example 19 on page 176, and Activity 48 on page 179 of MST124 Book B (Unit 5). Converting vectors from component form to magnitude and direction, and vice-versa, MST124 *Handbook*, page 42. Suggested revision: MST124 Unit 5, Subsection 6.4 Converting from component form to magnitude and direction, and vice-versa.
9	Example 1 on pages 231–232 of MST124 Book C (Unit 9). Matrix multiplication, MST124 *Handbook*, page 55. Suggested revision: MST124 Unit 9, Subsection 1.4 Matrix multiplication.
10	Example 4 on pages 255–256 of MST124 Book C (Unit 9). Matrix inverses and determinants, MST124 *Handbook*, page 56. Suggested revision: MST124 Unit 9, Section 3 The inverse of a matrix.
11	Example 4 on page 247 of MST124 Book B (Unit 6). Power functions, MST124 *Handbook*, page 44. Constant multiple rule and sum rule for derivatives, MST124 *Handbook*, page 44. Suggested revision: MST124 Unit 6, Section 2 Finding derivatives of simple functions.
12	Activity 8 on page 238 of MST124 Book B (Unit 6). Derivatives, MST124 *Handbook*, page 44. Differentiation rules, MST124 *Handbook*, pages 44 and 48. Suggested revision: MST124 Unit 6, Subsection 1.4 Derivatives, MST124 Unit 7, Subsection 2.1 Product rule.

Question	MST124 reference
13	Example 8 on page 266 of MST124 Book B (Unit 6). Stationary points, MST124 *Handbook*, page 45. First derivative test (for determining the nature of a stationary point), MST124 *Handbook*, page 45. Suggested revision: MST125 Unit 6, Subsection 4.2 Stationary points.
14	Example 3 on pages 19–20 of MST124 Book C (Unit 7). Quotient rule, MST124 *Handbook*, page 48. Suggested revision: MST124 Unit 7, Subsection 2.2 Quotient rule.
15	Example 12 on page 34 of MST124 Book C (Unit 7). Chain rule, MST124 *Handbook*, page 48. Derivative of a function of a linear expression, MST124 *Handbook*, page 48. Suggested revision: MST124 Unit 7, Subsection 2.3 Chain rule, MST124 Unit 7, Subsection 2.4 Differentiating functions of linear expressions.
16	Example 17 on page 167 of MST124 Book C (Unit 8). Integration by substitution, MST124 *Handbook*, page 52. Suggested revision: MST124 Unit 8, Section 3 Integration by substitution.
17	Example 2, page 112 and Example 3, page 117, of MST124 Book C (Unit 8). Signed areas and definite integrals, MST124 *Handbook*, page 50. Fundamental theorem of calculus, MST124 *Handbook*, page 51. Suggested revision: MST124 Unit 8, Section 1 Areas, signed areas and definite integrals, MST124 Unit 7, Subsection 4.2 Antiderivatives of power functions.
18	Example 12 on page 158 of MST124 Book C (Unit 8). Integration by substitution, MST124 *Handbook*, page 52. Suggested revision: MST124 Unit 8, Section 3 Integration by substitution.
19	Example 23 on page 178 and Examples 28 (pages 185–186) and 29 (page 186) of MST124 Book C (Unit 8). Integration by parts, MST124 *Handbook*, page 53. Suggested revision: MST124 Unit 8, Section 4 Integration by parts.
20	Activity 33 on page 176 of MST124 Unit 8. Integration by substitution, MST124 *Handbook*, page 52. Suggested revision: MST124 Unit 8, Subsection 3.5 Finding more complicated integrals by substitution.

Resources to prepare for MST125

The content of MST125 has been designed to follow on from that of MST124 *Essential mathematics 1*, so MST124 is the best preparation for MST125.

If you are already registered as an Open University student, you can make an early start on MST125 by looking at some units on the Mathematics and Statistics study site (learn2.open.ac.uk/site/S-MATHS) under DISCOVER ... *Discover your module.*

If you are not a registered student, or if you wish to consult some external resources, we recommend

- `http://fireflylectures.com/.`

- `http://www.mathcentre.ac.uk:8081/mathseg/`

However, if you have to make heavy use of these or other resources, we recommend that you consider studying MST124 instead.